

# U(1) axial as a force between neutrinos

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## Abstract

We show that when left and right handed neutrinos have a Majorana mass matrix, local gauge invariance produces a fifth force acting between chiral charges on neutrinos and quarks. The force is carried by a massless (or low mass) 1-spin gauge boson, we call an axiphoton. The force is caused by a U(1) axial gauge symmetry in the way as the electromagnetic force. We expect from renormalisation that the force constant,  $\alpha_a$  is about 1/60 of the electromagnetic force constant  $\alpha$ . We show that this force can explain dark energy. Our model predicts decaying right handed neutrinos in the eV-MeV range, and explain the heating of the solar corona. Finally we show that the Tajmar [32] experiment detecting a force due to a rotating superconductor, may be detection of our force.

## 1 Background

Physics to date, still has plenty of unsolved mysteries: What is the nature of dark energy and dark matter, why is the weak force left handed, what is the origin of parity violation, why is the mass of neutrinos so much smaller than the other particles. Despite of these mysteries the standard model of particle physics works absurdly well, leaving little particle physics experimental data unexplained.

In this paper we look at a Lagrangian containing both right and left handed Majorana neutrinos, we show that it has a  $U(1)$  axial symmetry which should generate an axial force when gauged. We estimate the strength of this force to be around 1/60 of the strength of the electromagnetic force from renormalisation theory, and show that it may be responsible for dark energy, and heating of the solar corona.

The bulk of experiments [11] seem to show an excess of low energy photon in quark gluon jets, we again show that our axial force predicts (in some models) additional bosons from these jets. These would not be photons but axiphotons but at high energy would look the same to many detector types.

We investigate the cosmic neutrino background in the presence of this force and show that it has the right properties to be dark energy.

Our model requires decaying right handed neutrinos with masses in the range  $1eV$  to a few  $MeV$ . We use the lightest to explain the heating of the solar corona.

Finally, no respectable force can be without experiment detection, an experiment [18] performed by Tajmar et al, detected a transient acceleration in an accelerometer near a rotating superconductor. We show how this could be a detection of the axial force. In normal matter a gas of neutrinos will shield the axial force, and normally will rotate with a rotating object. However in a rotating superconductor, the superconducting electrons do not rotate, we suggest that the weak force from the electron gas, will then freeze the neutrino gas in place, preventing it from screening the axial magnetic field due to the rotating nuclei. This will cause a net force on nearby objects.

## 2 Theory of the axial force

In modern field theory forces cannot be added to physics at will, but are generated from the underlying symmetries of the particles of nature. Consider a Dirac lagrangian with an arbitrary mass matrix  $M$ .

$$\mathcal{L} = i\bar{\phi}\gamma^\mu\partial_\mu\phi - \bar{\phi}M\phi \quad (1)$$

In this lagrangian we can represent standard dirac masses, as  $M = mI$  but we can also represent Majorana masses. Let us use the Weyl basis for the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \quad (2)$$

With the Pauli Matrices  $\sigma_i$  as;

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3)$$

The charge conjugation operator is then  $C = i\gamma^2[\ ]^*$ .

Now if we take an off diagonal mass matrix,

$$M = \begin{pmatrix} 0 & Rm_r \\ Rm_l & 0 \end{pmatrix} \quad (4)$$

Where  $R$  is some yet unknown submatrix.

In the Weyl basis we may write  $\phi$  as

$$\phi = \begin{pmatrix} \phi_r \\ \phi_l \end{pmatrix}. \quad (5)$$

We find that the Dirac equation,

$$i\gamma^i \partial_i \phi - M\phi = 0, \quad (6)$$

decouples into two independent equations for two independent fields.

$$i\frac{\partial\phi_r}{\partial t} + i\sigma_j \frac{\partial\phi_r}{\partial x_j} - m_r R\phi_r = 0, \quad i\frac{\partial\phi_l}{\partial t} - i\sigma_j \frac{\partial\phi_l}{\partial x_j} - m_l R\phi_l = 0. \quad (7)$$

Thus with our off diagonal mass matrix we can represent both standard left hand neutrino's and the proposed heavy right handed neutrinos. Taking the left hand equation and multiplying by the opposite momentum operator, we have:

$$-\frac{\partial^2\phi_l}{\partial t^2} + \frac{\partial^2\phi_l}{\partial x_j^2} - \left( i\frac{\partial}{\partial t} + i\sigma_j \frac{\partial}{\partial x_j} \right) m_l R\phi_l = 0 \quad (8)$$

Comparing with the Klein Gordon equation, implies:

$$iR\frac{\partial\phi_l}{\partial t} - i\sigma_j R\frac{\partial\phi_l}{\partial x_j} - m_l\phi_l = 0 \quad (9)$$

Thus we require that,  $R = R^{-1}$  and  $R\sigma_i = -\sigma_i R$ . This is not possible with just two complex components and so we will need to extend each of  $\phi_l$  and  $\phi_r$  to four complex components.

Returning to our Langragran, equation(1), we can see that the momentum part of the Langragian, is invariant under the transformation:

$$\phi \rightarrow e^{-i\lambda} \phi \quad (10)$$

which we call a vector transformation. It is also invariant under:

$$\phi \rightarrow e^{-i\lambda\gamma^5} \phi \quad (11)$$

which we call an axial transformation. In the above  $\lambda$  is an arbitray phase change, and  $\gamma^5$  is the 5th dirac matrix, the chirality operator.

Now in the case of the electron, we have a dirac mass,  $M = mI$ , and we see the mass term is not invariant under the axial transformation. However consider the case of a Majorana neutrino, with the above off diagonal mass matrix. This is invariant under the axial transformation. Under this transformation,

$$\bar{\phi} = \phi^\dagger \gamma^0 \rightarrow \phi^\dagger e^{i\lambda\gamma^5} \gamma^0. \quad (12)$$

So that,

$$\mathcal{L}_m = \bar{\phi} M \phi \rightarrow \phi^\dagger e^{i\lambda\gamma^5} \gamma^0 M e^{-i\lambda\gamma^5} \phi. \quad (13)$$

This will be invariant provided that  $\gamma^0 M$  commutes with  $\gamma^5$ , i.e. if

$$[\gamma^0 M, \gamma^5] = \gamma^0 M \gamma^5 - \gamma^5 \gamma^0 M = 0. \quad (14)$$

Which is true for our off diagonal Majorana mass matrix but not for standard diagonal dirac mass.

In fact it is well known Majorana neutrinos do not support any  $U(1)$  vector charges. But it seems possible that they support a axial charge. If we promote our above global axial transformation to a local transformation then we force the majorana neutrinos to be coupled to a  $U(1)$  spin-1 massless field.

### 3 The axial current, and charge assignments

In electroweak theory the electromagnetic current, the source of the E.M. field can be written:

$$j_\mu^{em} = -\bar{e}\gamma_\mu e = -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R. \quad (15)$$

We can also form an axial current:

$$j_\mu^a = -\bar{e}\gamma^5\gamma_\mu e = -\bar{e}_L\gamma_\mu e_L + \bar{e}_R\gamma_\mu e_R. \quad (16)$$

Could this axial charge also be the source of a force. In the case of an electron most definitely not, the QED axial anomaly will cause the axial current to be non conserved. However consider the case of a neutrino with majorana mass, any vector charge or magnetic moments are foribbiden. In this case we can take the two field of light left handed neutrinos and right handed anti-neutrinos, and of heavy right handed neutrino and left handed anti-neutrino. We apply a phase difference between the two fields at every point in space-time, i.e. a local guage transformation, which will source a spin-1 field. Following the usual analysis of local guage transformations, we are forced to introduce an axial field with 4-potential  $B_\mu$ , and field strength  $G_{\mu\nu}$  where,

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (17)$$

We then have a Langragian of form,

$$\mathcal{L} = [i\bar{\psi}\gamma^\mu\partial_\mu\psi - m_l\psi_l^c\psi_l - m_r\psi_r^c\psi_r] - \frac{1}{4}G^{\mu\nu}G_{\mu\nu} - q(\bar{\psi}\gamma^5\gamma^\mu\psi)B_\mu. \quad (18)$$

We may form the axial charge density,

$$\rho = -q\bar{\phi}\gamma^5\gamma^0\phi = q(\phi_l^\dagger\phi_l - \phi_r^\dagger\phi_r) \quad (19)$$

and a current in the  $i^{th}$ -direction,

$$J_i = -q\bar{\phi}\gamma^5\gamma^i\phi = -q(\phi_l^\dagger\sigma_i\phi_l + \phi_r^\dagger\sigma_i\phi_r) \quad (20)$$

Let us assign a -1 axial charge to the neutrino. Then the anti-neutrino has a +1 axial charge. If right handed neutrinos exist then they have +1

axial charge, and cannot convert to left hand neutrinos. Then neutrino dirac masses are forbidden and masses of the left and right hand neutrinos can be different.

If the axial charge sources a field, it must be locally conserved everywhere, hence from the fact a  $W^+$  decays to an positron and an electron neutrino, the  $W^+$  must have an axial charge of  $-1$ . Similarly the  $W^-$  will have a charge of  $+1$ , and the  $Z$  and photon no charge. Finally because  $W^+ \rightarrow u + \bar{d}$ , the charge on the up type quarks must be  $-1$  less than the charge on the down type quarks, and all the generations must have the same charge values. We do not yet know the absolute value charge on the quarks. We find that particles of opposite spin have opposite axial charge, and anti-particles have the opposite axial charge to particles. We show in the next section that quarks are actually majorana states and have low and high mass eigenstates, with the left and right handed states of the same mass haveing the same (not the opposite) axial charge. The charges on the u and d quarks can be derived from the requirement of anomaly calculation, for the minute we will put,  $Q_a(u) = -1/2$  and  $Q_a(d) = +1/2$ . In summary,

Particle	$u_L$	$d_L$	$u_R$	$d_R$	$e^-$	$\nu_L$	$N_r$	$W^-$	$Z$	$W^+$	$\gamma$
EM charge	2/3	-1/3	2/3	-1/3	-1	0	0	-1	0	+1	0
Axial charge	-1/2	+1/2	-1/2	+1/2	0	-1	+1	+1	0	-1	0
anti-Particle	$\bar{u}_L$	$\bar{d}_L$	$\bar{u}_R$	$\bar{d}_R$	$\bar{e}^+$	$\bar{\nu}_L$	$\bar{N}_r$				
EM charge	-2/3	+1/3	-2/3	+1/3	+1	0	0				
Axial charge	+1/2	-1/2	+1/2	-1/2	0	-1	+1				

Because the weak force carriers have both a electric and a axial charge, they are forced to be left-hand and its  $(V - A)$  vector minus axial nature is automatically enforced by conservation of charge. One could also have a right handed weak force, but it would be completely decoupled from the ordinary weak force, and its force carriers need not have the same mass as the  $W$  and  $Z$ .

This is not the first time a axial force has been suggested, and indeed a paper by L.M. Slad [7], we found while writing this, introduces essentially the same force from a group theory perspective. Slad demonstrated that the weak force is not invariant under the complete Lorentz group without having an additional axial force. His model added an axial force complete with a high mass right handed weak force, and right-handed neutrinos. He did not however take the time to investigate the phemonology of the axial force to any great depth.

Returning to our particle table. We must now check our charge assignments for anomalies, in Slads paper he assigned opposite axial charge to two generations of quarks and leptons, and zero axial charge to another generations. Although this will cancel any anomalies, it would not give the correct

weak force behaviour between quark generations, one generation would be right handed in its decay, and another would not feel the weak force at all. Instead we will need another copy of each generation of quarks, with opposite axial charge, that is heavy enough not to have been seen as yet.

Anomalies cancellation requires that:

$$\sum Q_a = 0 \quad , \quad \sum (Q_a)^3 = 0 \quad (21)$$

This is clearly true with the above charge assignments if you can include both the left and right side states, however, we require anomalies to cancel within the left handed and right handed states separately. The neutrino is cancelled provided with have right handed states in addition to the left handed states.

We have additional anomalies in electric and axial charges for the quarks which carry both:

$$\sum_q (Q_a)^2 Q_e = 0 \quad , \quad \sum_q (Q_e)^2 Q_a = 0 \quad (22)$$

The second sum above cancels when we introduce the heavy teraquark states with the opposite axial charge. The first sum is easierly solved to give transcendental (i.e. not integer or fractional) axial charges on the quarks. We take the root where that u and d quarks have opposite charge then:

$$Q_a(u) = 1 - \sqrt{2} \approx -0.4142 \quad , \quad Q_a(d) = 2 - \sqrt{2} \approx 0.5858 \quad (23)$$

This gives the proton an axial charge of  $-0.2426$  and the neutron of  $0.7574$ , slightly over -3 times the proton's axial charge. With transcendental charges on the quarks, baryon number must be absolutely conserved, this is good since proton decay has still never be detected and this lack is problem for grand unified theories. However we must allow for some way of producing the baryon number asymmetry in the universe. This could be done in the mirror matter model by transferring axial charge from our world to the mirror world. E.g. by  $2D + U \rightarrow 2d' + u' + \dots$  or  $2\bar{d} + \bar{u} + \dots$ , a teraneutron decaying either to antineutron or a mirror neutron.

An alternative to transcendental charges and mirror matter, comes in E6 and trinification models, which contain three colors of an extra h-quark per generation. The h-quarks generally have the same charge as the d, but are heavy enough to be undetected. Adding the h-quarks to our anomalies sum above, gives

$$Q_a(u) = -\frac{1}{2}, \quad Q_a(d) = \frac{1}{2} \quad (24)$$

which seems more attractive. Protons are still absolute stable, but the above charges allow deuteron and baryon-pairs to decay. This keeps matter stable, (deuteron decay will be of very high order, due to it being a six quark process and so very slow) but allows for Baryon number asymmetry to be created in the big bang. We much prefer the half integer baryonic charges as physical and so we continue to use these for the rest of the paper.

## 4 A problem with quarks

Consider the quark field, from our above discussion of charges on the quarks, at least one the up and down quark states must feel both the electric force and our axial force. But the fields are generated by the invariance of lagrangian, which depends on whether the mass type is dirac or majorana. We do not seem to be allowed to have a massive fermion field invariant under both axial and vector transformation. Also if the up and down quarks have opposite axial charge, then the current we get is  $\bar{q}\gamma^5\gamma^\mu t_{isospin}q$  which is the well known PCAC, partially (and therefore not completely) conserved axial current of QCD. In addition to this, it is known that a proton or a neutron can change its spin freely, but if its transverse spin changes its axial charge will change, violating axial charge conservation. Is this the end of our axial force? No we have a trick up our sleeves.

In order for the axial force to coexist with the electric force for quarks, we introduce a trick we call majorana color. QCD is invariant under  $SU(3)$  and not  $U(3)$ , thus the total amount of color doesn't matter, and we can produce white states in three ways,

$$white = r + g + b = \bar{r} + \bar{g} + \bar{b} = color + anticolor \quad (25)$$

Normally in QCD we assign each quark a color (one of red, green or blue) and each antiquark an anticolor one of  $\bar{r}$ ,  $\bar{g}$ ,  $\bar{b}$ . Instead of this lets assign different color types to the left and right handed quark states.

$$\begin{aligned} q_l &\in (r, g, b) \\ q_r &\in (\bar{r}\bar{g}, \bar{g}\bar{b}, \bar{r}\bar{b}) \\ \bar{q}_r &\in (\bar{r}, \bar{g}, \bar{b}) \\ \bar{q}_l &\in (rg, gb, rb) \end{aligned}$$

Despite the change, our usual 3 quark baryon states, and meson states are still white. e.g.

$$proton(left) = u(left, red)u(left, green)d(right, anti - red + anti - green)$$

$$\pi^0 = \frac{1}{\sqrt{2}} [(u(left, red)\bar{u}(right, gb)) + (d(left, red)\bar{d}(right, gb))]$$

Now we can generate our axial charges depending on the color state of the quarks as well as there spin and isospin, we can take the same axial charge for both the left and right quark states. The price we pay for this, is that we require heavy copies of the quarks with opposite axial charges. These states are similar the teraquarks in S. Glashow's sinister standard model [8] extension, and are also found in littlest Higg's models [14]. Unlike in the sinister model, we have not requirement that these states be stable.

This is a good thing, since Fargion and Khlopov [9] showed that these would be at odds with astrophysical observations. In order to fit observations our teraquark could either form a stable tera neutron, a dark matter candidate, which would require careful tuning of the teraquark masses. Alternatively the tera-baryons could decay into normal low mass states.

## 5 Death to the axion

With majorana color left and right handed quarks now in different representations of the color  $SU(3)$  group, we have a new QCD Langragian: Where A and B represent the two different representaions the quarks can be in.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\theta + \mathcal{L}_m \quad (26)$$

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i \sum_f \sum_i \sum_j \bar{\psi}_{Afi} \gamma^\mu \left( \delta_{ij} \partial_\mu + ig_s \sum_{a=1}^8 G_\mu^a \lambda_{a,ij} \right) \psi_{A fj} \\ & + i \sum_f \sum_i \sum_j \bar{\psi}_{Bfi} \gamma^\mu \left( \delta_{ij} \partial_\mu + ig_s \sum_{a=1}^8 G_\mu^a \lambda'_{a,ij} \right) \psi_{B fj}, \\ \mathcal{L}_\theta = & \frac{\theta}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} \end{aligned} \quad (27)$$

The field strength tensor is

$$F_{\mu\nu}^a = \partial^\mu G_\nu^a - \partial^\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c \quad (28)$$

We form the alternate representation of the color group. With the usual Gell Man Matrices the structure constants are:

$$f^{123} = 2, f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = 1, f^{458} = f^{678} = \sqrt{3} \quad (29)$$

Since the field strength tensor depends upon the structure constants, our alternate representation must have the same structure constants in the same permutations. The only possiblity is to swap two pairs of the matrices so that:

$$\lambda'^4 = \lambda'^6, \quad \lambda'^5 = \lambda'^7, \quad \lambda'^6 = \lambda'^4, \quad \lambda'^7 = \lambda'^5 \quad (30)$$

Its possible to prove that the above permutation cannot be made with an unitary transform. Thus the permutation is not itself a member of  $SU(3)$ , and therefore cannot be guaged away.

Now we can introduce the quark masses as,

$$\mathcal{L}_m = \sum_f \sum_i \sum_j \sum_{x=A,B} \sum_{y=A,B} \bar{\psi}_{fj} M_{fxy} \psi_{fi} \quad (31)$$

In order for the both the electric and axial forces to invariant we require, M to be diagonal in the up, down flavor space, and off-diagonal in x and y. The color group then becomes  $SU(3)_L \times SU(3)_R$  and the axial part of QCD is not invariant. We may then write the masses as:

$$\mathcal{L}_m = m_l \psi_{Af}^c \psi_{Bf} + m_h \psi_{Bf}^c \psi_{Af} \quad (32)$$

We now have both light and heavy quark states, with opposite axial charges. The quarks are now free to change spin without breaking axial charge conservation. This mixing thus leads to two pairs of Dirac massed quarks.

Finally let look at the  $\theta$  term in the Langragian, this is term is CP violating. But experiments, for example measurements of the electric dipole moment of the neutron are zero to very high accuracy, showing that QCD preserved CP. This lead Pecci and Quinn to introduce a psuedoscalar axion, in order to remove the possibility of this term in the Langragian. We may see however that the  $\tilde{F}F$  term is not invariant under our axial transformation of section 1.

$$\psi \rightarrow e^{-i\lambda\gamma^5} \psi \Rightarrow \theta g^2 F \tilde{F} / 32\pi^2 \rightarrow (\theta - \lambda) g^2 F \tilde{F} / 32\pi^2 \quad (33)$$

Thus the axial force automatically bans the theta term from the Langragian.

## 6 Embedding the SM in $E_6$ with teraquarks

We saw earlier that we need to give quarks two representations of the color group with opposite axial charges both to cancel anomialies and to allow nucleons to freely rotate. Can with find a GUT which contains these particles. In fact we can:  $E_6$  can contain exactly these states with:

$$27 \rightarrow 10 + \bar{5} + 6 + \bar{6} \quad (34)$$

and particle asignment:	$d_{rL}$	$u_{rL}$	$D_r$	$D_g$	$D_b$	$u_{rgR}$	$d_{rgR}$
	$d_{gL}$	$d_{gL}$	$U_r$	$U_g$	$U_b$	$e_L^\dagger$	$u_{gbR}$
	$d_{bL}$	$d_{bL}$	$e_L^-$	$\nu_L$	$\bar{D}_{br}$	$u_{brR}$	$d_{brR}$
			$\bar{D}_{rg}$	$\bar{D}_{gb}$	$\bar{D}_{br}$		
			$\bar{U}_{rg}$	$\bar{U}_{gb}$	$\bar{U}_{br}$		

$E_6$  is a chiral group and our version here, only contains a left handed neutrino, and the left handed weak force. To complete the theory, we need to add mirror matter so the complete group is  $E_6 \otimes E'_6$ . If, as above, we embed the SM particles with both handness, and the teraquarks with left and anti-right state. We may firstly cancel the electric charges on the teraquarks with the antiteraquarks, and secondly explain there high mass by the fact that they are not chiral.

We don't have enough states to produce the needed right handed neutrino nor do we have a right handed weak force. The above embedding is a low energy version where spontaneous left-right symmetry breaking has already occurred. We can suppose that our chiral  $E_6 \otimes E'_6$  model, is produced by the breaking of a string inspired  $E_8 \otimes E'_8$  model.  $E_8$  is non-chiral so the breaking left-right symmetry should take place during  $E_8 \rightarrow E_6$ . In fact if we start with the smaller  $E_7$  model, which is non-chiral, and so presumably left-right symmetric. We can break it as,

$$E_7(56) \rightarrow E_6(26) \oplus \bar{E}_6(26) \oplus \nu(1) \oplus \bar{\nu}(1)$$

Which give us precisely enough particles for a matter and antimatter representation of the  $E_6$  and plus extra two neutrino states. We still require an 8-state majorana neutrino. To do this we might introduce a mirror matter  $E_6$  group. The axial force is then shared between matter and mirror matter. The neutrino is shared between the matter and mirror matter groups, to produce a single 8 state object as described below. This has the added advantage that the mirror matter model does not suffer 3 extra stable low energy neutrinos which would upset big-bang nucleosynthesis constraints. This model fits perfectly into string theories favourite symmetry,  $E_8 \otimes E'_8$ .

As we saw in the last section, we need the teraquarks in order to cancel the axial anomalies between the electric and axial forces, and to allow nucleons to change spin freely. And we guessed that the teraquarks would gain a very high mass in order to hide them from experiment. But perhaps there are low mass after all, and have been already seen. The quark model has a hidden defect compared to experiment. There seem to be more baryons and mesons than standard QCD theory can explain. The lightest of these extra particles is the  $\sigma(555)$  meson. Its a state like a pion (isospin triplet, a set with positive, negative and neutral particles), except that its scalar  $J^p = (O^+)$ , whereas the pion is pseudoscalar  $J^p = (O^-)$ . It decays so rapidly its width is bigger than its mass. QCD theorists have tried to explain it as a four quark state, but recently experiments [38] have shown it to have much too large a photon coupling for that to be true. Although vanishingly short lived, the  $\sigma$  meson is important, nucleon-nucleon binding just wouldn't be strong enough to hold nuclei together without it. Our hypothesis then is that the  $\sigma$  is made of one ordinary quark and one anti-teraquark. Because the teraquark has isospin already in a sense opposite to a quark, taking its anti-particle, reverses parity twice, so the combination is a positive parity state. In fact M & S Ishida [37], have already suggested that doubling the number of quark states provides a fit for the  $\sigma$  meson as well as many of the other unexplained quickly decaying meson and baryon resonances. Would having these extra quark states being of low mass be compatible with existing measurements of quark numbers. We think its possible, the standard method of counting the number of quarks belows a

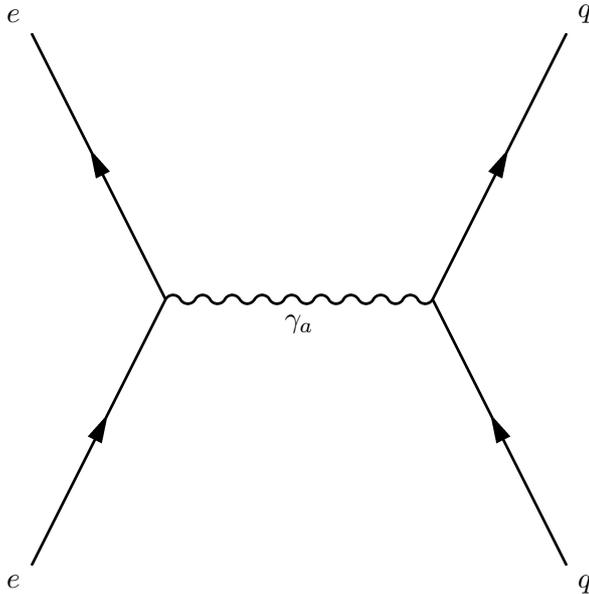


Figure 1: Quark Pair Production

certain mass is by using the ratio of hadron to muon production.

$$R = \frac{\sigma(e^- + e^+ \rightarrow \text{hadrons})}{\sigma(e^- + e^+ \rightarrow \mu^+ + \mu^-)} = 3 \sum_f Q_f^2$$

By naive reasoning adding the teraquarks would double R which would disagree with measurements, however since the final state always contains normal quarks, the cross section due to teraquarks should be reduced by a factor of the coupling constant of the force which makes them decay as shown in the figures below:

All that remains so to explain why the teraquarks are so short lived. To do this, we take from  $E_6$  two extra octets of axially charged gluons. If we give all the quark and teraquark states axial charges of either plus or minus 1/2. We naturally find a standard octet of gluon acting between quark states (and also teraquark states). Plus a positive and negatively axially charged octet for gluons which transform teraquarks to standard quarks. These should also be massless and have the same order of force strength as the standard gluons. The decay of the sigma is then mediated by the axially charge gluon octet, transforming the teraquark to a standard quark, and releasing another quark pair. Would having axially charged gluon states have an noticeable effect on the standard model calculations. Clearly yes. Firstly they lead to the emission of extra axiphotons from quark jets. If these are observed as photons, then such a photon excess has indeed been observed [11]. The second effect would lead to an increased neutral current cross-section for neutrino scattering, which has not been observed. However

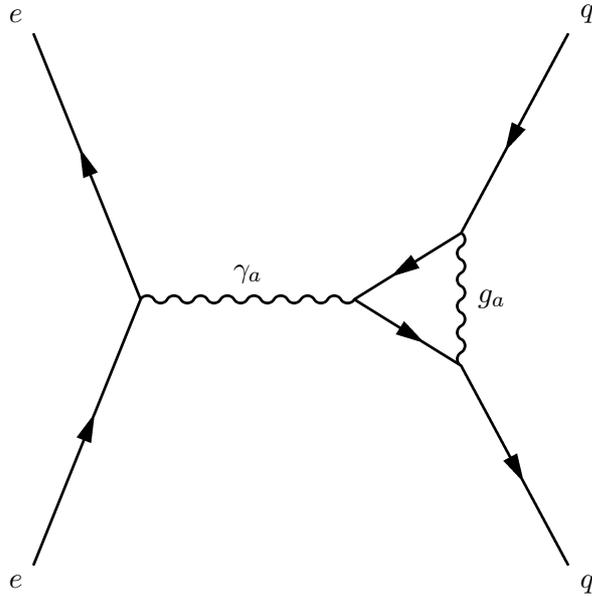


Figure 2: Quark Pair Production via teraquarks

for most states the effect the positive and negatively axially charged gluons should cancel out.

We must admit that this  $E6$  model and speculation about teraquarks and the sigma mesons, is speculation unsupported by detailed calculations or even proper checks of the group theory and anomalies. We hope that should it prove to be baseless that it will not detract from our neutrino and axial force model. We however also hope then that QCD and meson experimentists, will that our suggestion seriously enough to calculate it ramifications and to test it in the future.

## 7 Dense nuclear matter, Fermi degeneracy and axial symmetry restoration

Above we showed how protons and neutron could continue to spin freely, even given a conversed axial force. To do this we had to introduce teraquark states with the opposite axial charge. But we now find another problem: spin up and spin down neutrons have the same axial charge. In order to cancel this out, we are going to need a lot of neutrinos to be present in ordinary matter, and even more in dense nuclear matter. Because neutrinos are light fermion they will introduce a lot of dengency pressure in matter. So much in fact that neutron star would be impossible. (The same would also apply to strange quark matter). Astrophysical observations of pulsars seem well modelled as neutron stars. So for our theory to be tenable, neutron stars had better exist. Models of meson masses, seem to show that interaction

between quarks regains its invariance under axial U(1) for color and flavour at high energy, a few GeV [26]. In order match this we will introduce a new see-saw mass into our quark mass model above.

$$\mathcal{L}_m = m_3 [\bar{\psi}_{Af}\psi_{Bf} + \bar{\psi}_{Bf}\psi_{Af}] + m_+ [\psi_{Afl}^c\psi_{Afl} + \psi_{Bfr}^c\psi_{Bfr}] \quad (35)$$

$$+ m_- [\psi_{Afr}^c\psi_{Afr} + \psi_{Bfl}^c\psi_{Bfl}] \quad (36)$$

The above mass matrix may seem a bit ad hoc, but in fact it can be generated simply by having one axially and electrically neutral color triplet Higgs boson, and also one axial positive and one axial negative Higgs boson. Now we assume that in ordinary space, the positive mass eigenvalues of the above give the mass as in the presiding section. As the density of nuclear matter increases the density of axially charged Higgs' will increase much faster than the colored Higgs, due to the overall color neutrality of nuclear matter. As the density (or energy) increases, we eventually reach a point where, the low mass eigenvalues of the above become Dirac masses rather than color Majorana masses. Physically the following reaction can occur: Whereby pairs of neutrons in nuclear matter reverse their axial charge while keeping the same spin, by changing mass state.

$$2\nu + 2n_l^{a=+\frac{1}{2}} \cdot 2n_r^{a=+\frac{1}{2}} \rightarrow 2n_l^{a=+\frac{1}{2}} \cdot 2n_r^{a=-\frac{1}{2}} + \gamma_a \quad (37)$$

The energetics of the above, come only slightly from the repulsion of the axial charges, but mainly from the alteration of the masses of quarks by the various Higgs fields. With the above mass type and the correct scaling of each of the mass terms, we have changed nuclear matter from being heavily axially charged, to being only slightly charged, allowing neutrons stars to exist once more. Without knowing the form of the Higgs potentials it is difficult to perform the detailed calculations to show that the above construction can correctly change the quark mass type from color majorana to dirac in dense matter, while keeping the other state hidden from researchers to date. If the mass of the u and d quarks varies at high density such effects would show up in the masses of nuclei. It is known that the number of protons - neutrons is a strong factor in the masses of nuclei, and that paired protons or paired neutrons give a strong increase in nuclear binding energy. It would be interesting to see, how well our above models help improve estimates of nuclear binding energies.

## 8 Fermi degeneracy and right handed neutrinos

Back on earth, degeneracy pressure is a problem to. Water has two uncancelled proton axi charges, will need some like  $Av/18$  neutrinos per gram to cancel its axial charge,  $Av$  being Avagadro's number, or about  $6.022 \times 10^{23}$

. Heavy elements, eg.  $U - 238$  will need more, about  $73Av/238$ . Since degeneracy pressure scales as  $1/m$ , this is a real problem for meV massed neutrinos. The formula for the Fermi energy of a N bound fermions of mass m in a volume V is.

$$E_f = \frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{2/3} \quad (38)$$

$$E_f(ev) = 1.865 * 10^{-9} (N(cm^{-3}))^{2/3} / m(ev/c^2) \quad (39)$$

At as mass of  $60meV/c^2$  we end up with a Fermi energy of about  $650MeV$  for neutrinos in uranium. This simply isn't on, neutrinos with energy above  $0.5MeV$  or so, would cause inverse beta (neutrino capture) decay of nuclei, which isn't observed. Naturally with multiple neutrino types, all the energy levels of each type will be filled up to the same energy. Solar and Atmospheric neutrino mixing gives a mass splitting of  $\Delta m_{12} = 8.8meV$  and  $\Delta m_{13} = 49meV$  [13] so the heaviest neutrino has a mass in the range  $0.05eV/c^2 < m_\mu < 0.7eV/c^2$ . At  $0.7eV$  (too high as mass cosmologically) we get a Fermi energy of  $55MeV$  in uranium, still too high. In order to fit with observations, we are going to need neutrinos in much higher mass range. And to keep stars (especially white drawfs) the same as in current theories, we will in fact need a neutrino heavier than the electrons. These neutrinos had better be sterile because they haven't been observed in weak decays. Thus we are lead to needing several sterile right handed neutrinos states in the few  $eV$  to few  $MeV$  range.

Normally the see-saw mechanism gives right handed neutrinos masses near GUT levels. However low mass right handed neutrinos are possible, if not particularly natural. In E6 GUTs [35] low mass right handed neutrinos are natural because they have additional neutral singlet states that can have the high mass range. Astrophysically and experimentally low mass right handed neutrinos become very useful. A (RH) neutrino in the few  $KeV$  mass range, seems like the best explanation of neutron star kick [15]. Neutron stars seem gain high speeds of up to  $3000Km/sec$  out of there forming supernova, which is mystery, that a asymmetric emission of sterile neutrinos in the  $KeV$  range can solve. The LSND experiment showed reasonably strong evidence for neutrino oscillations  $\nu_\mu \rightarrow \nu_e$  with mass splitting in the few  $eV$  range. This can be explained [16] by the mechanism of a sterile right handed neutrino with a mass of a few  $eV$  which latter decays to an electron neutrino. A  $KeV$  RH neutrinos seems good for keeping teresterial matter neutral, but in the case of white drawfs we will need a state in the  $MeV$  range, more massive than the electron, again there are fragments of evidence for such a state. In the center of our galaxy, Sagittarius A\*, (a missnomer, its a supermassive black hole and not A star) there is evidence for an excess of positions as measured by the  $0.511MeV$  gamma ray emission line. A neutrino with a mass of a  $2 - 20MeV$ , would explains this mystery [17].

Another possible sighting was made by Karmen group [19], which detected a rare slower moving Pion decay product, consistent with a  $39MeV$  neutrino.

These states could form dark matter, but in our model as we calculate in the next section, they decay rather rapidly to standard left handed neutrinos states, the left handed neutrinos being themselves stable. Only an  $eV$  range RH neutrino will be in equilibrium around the time of nucleosynthesis in the big bang and so one extra state will be tolerable in terms of upsetting the calculation of the amount of primordial elements.

We need only the lightest of these RH neutrino state to get the needed Fermi energy levels in say water or uranium. Let us take for example a  $10keV$  right handed neutrino. And let the mass of the heaviest left hander be  $0.1eV$  this will appear in matter when its axial charge density is high enough to give our 3 left hand neutrinos a Fermi energy greater than  $10KeV$ , i.e. when

$$N > \left( \frac{M_r M_l}{1.865 * 10^{-9}} \right)^{3/2} \approx 4 * 10^{17} cm^{-3} \quad (40)$$

The requirement of axial charge neutrality easily calculates the number densities of neutrinos in matter:

We assume charges of  $\pm \frac{1}{2}$  on the protons and neutrinos and no return to dirac masses in the heavy elements. All figures are per  $cm^3$

Substance	$\nu$ No. density	$E_m(\nu_r)$ ( $MJ/cm^3$ )	$E_f(\nu_l)$ $KeV$	$E_f(\nu_r)$ $eV$
Dry Air	$1.0 * 10^{17}$	0	4	n/a
Water	$-3.3 * 10^{22}$	52	10	190
Uranium	$3.4 * 10^{24}$	4900	10	4200
Bismuth	$41.77 * 10^{24}$	2800	10	2700
NaCl	$2.7 * 10^{22}$	43	10	170
Pyrex Glass	$1.16 * 10^{21}$	1.7	10	21
Palladium	$2.05 * 10^{24}$	3300	10	3000

The above makes for interesting chemistry. Most solids and liquids will contain a degenerate gas of neutrinos, including the heavier right handed states. Such material would be excellent conductors of neutrinos or should we call it axitricity (c.f. electricity). In such material the axial force will be strongly screened. Water, plastic and organic matter would have the opposite charge carriers than stone and metal. Might the neutrinos contribute to heat flow in matter? If so, experiment might detect easier flow of heat between objects with the same type of charge carrier. Air is an interesting example, at zero humidity we have the above result, however around 2% humidity we need almost no neutrinos, and at 4% humidity (near saturation) we need around  $1.0 * 10^{17}$  antineutrinos. Anecdotally many people say that it feels far colder in damp air than dry air at the same temperature. Could neutrino carried heat flow be reason? It seems a hard step to take to believe in all these neutrinos in normal matter, just to neutralise our new force, but the question is, does it contradict with any known experiments? The particles

at most danger of causing a contradiction are the right handed neutrinos, these normally would decay to left handed neutrinos (we will calculate the rate later), but are supported by fermi degeneracy pressure in most materials. During a change of state of a material, they may be liberated, or may in fact be required to be created by inverse decay.

One example is desolving heavy elements in water. The heavy substance will contain anti-neutrinos while the water neutrinos, these could annihilate, however the cross section (see later) is small. The neutrinos will no longer be bound by a potential and will travel a fair distance before they decay, annihilate or find a new home. When however we extract the elements from water, we will need to supply the extra binding energy. The need neutrinos will either be sucked in from outside or pair produced on the spot (causing local cooling).

Consider cloud formation, in the damp air we may not need right handed neutrino states, but when water droplets form, the needed neutrino density is such that right hand states will be needed, in these are formed in situ, this would cause a cooling effect on the droplets. Conversely during boiling right handed neutrinos might be released and if they decay they deposit there energy into the surrounding, they might however travel some fair distance before doing so. The decay would liberate an axiphoton in the  $eV$  to  $KeV$  range depending on the mass of the right handed neutrinos. Axiphotons however cannot interact with electrons only nuclei. So despite there high energy the axiphotons would not be ionizing. Could such processes be carrying on in the earth without us having noticed? We will, at present, leave the matter for further speculation.

One final speculation here though. Consider the electrolysis of water, where in, the hydrogen is absorbed into a heavy metal, (e.g. palladium which can hold many times its volume of hydrogen gas.) Neutrinos would be liberated in process, and while normally they would be expelled by the repulsion of protons in water, if the electrolyte was mostly heavy water, and the surrounding materials were mostly repellent of neutrinos, then  $\bar{\nu}_r$  would decay in the heavy water releasing they mass energy. Could this be the explanation (does it need one?) of excess energy in electroysis of hydrogen absorbing metals, previously thought of as cold fusion. Many researchers have found heat excess in such experiments, but with minimum detection of neutrons, gamma rays and fusion products. As well known, the magnitude of the electrical repulsion between nuclei is so high that no known mechanism could get deuterium to fuse at ordinary temperatures and pressure. So theoretically cold fusion makes no sense. Perhaps the decay of light right handed neutrinos is the correct explanation of the heat excess in such experiments. If this so, then it is hydrogen not deuterium that needs to be implanted into the metal, but the heavy water is needed to form a cavity in which the neutrino is trapped before it can decay. This then leads to both an energy storage system and a primary energy generation

mechanism using the earths heavy element mineral deposits as fuel sources.

## 9 How strong is this axial force

We can from an estimate of the strength of the axial force, by renormalisation theory, if at grand unification scale  $E_u \approx 10^{27}ev$ , the strengths of all the forces are the same. As well known charge particle pairs screen the electric field as:

$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 - \frac{\alpha(\mu_0)}{6\pi^2} \ln \frac{\mu}{\mu_0}} \quad (41)$$

In the same way, axially charged particle will screen the axial charge. The lightest of these, the neutrino is around a billion times lighter than the electron. Thus the screening of axial charge is much stronger. Then reusing the above equation, for the axial force we predict the low energy force strength as around.

$$\frac{\alpha_a(0)}{\alpha(0)} \approx \ln \left( \frac{m_e}{m_\nu} \right) \approx \frac{1}{21} \quad (42)$$

Now if we include three generations of neutrinos in the range  $1meV$  to  $1eV$  the force strength further reduces to around  $1/60$  of alpha, this is still very high but there will be a second shielding effect.

As well as shielding from virtual neutrinos, physical neutrinos will further shield the axial force. Given a thermalised neutrino plasma with temperature  $T$ , number density  $n$ , the screening effect should be analogous to the debye screening of the electric force. Let  $q_a$  be the axial charge on the neutrino, and  $\epsilon_a$  the axial permittivity of free space. Then

$$\frac{q_a^2}{\epsilon_a} = \frac{e^2 \alpha_a}{\epsilon_0 \alpha} \quad (43)$$

The Debye length will be,

$$\lambda_d^{-2} = \sum_a \frac{n_a q_a^2}{\epsilon_a kT} \quad (44)$$

and the plasma frequency will be,

$$\omega_p^2 = \frac{n q_a^2}{m_\nu \epsilon_a}. \quad (45)$$

Standard estimate of the neutrino background [10] are  $T \approx 1.9K$ ,  $n \approx 336cm^{-3}$ , and  $m_\nu \approx 1meV$ , with  $\alpha = 20\alpha_a$  this gives.

$$\lambda_d = 7mm \quad , \quad \omega_p = 80KHz \quad (46)$$

The above gives an estimate of the strength of the axial force in empty space. On earth we might expect the neutrino background to be much higher reducing the strength further, in air at ground level we have estimated  $10^{17}$  neutrinos  $cm^{-3}$  and  $T \approx 300K$ , giving,

$$\lambda_d = 5nm \quad , \quad \omega_p = 2.6 \times 10^{16} Hz \quad (47)$$

Clearly the (primary) effects of the axial force will be so strongly shielded in air as to be unnoticeable. In a experimenters vacuum we would still expect a strong neutrino background to leak in from the surroundings so here too the force would be strongly shielded. Thus we would not expect to measure the effects of the axi-electric force in experiments.

## 10 Neutrino interactions, and classical action of the axial force

When we started worked on the axial force, the question that inspired us, was, "What happens if you overtake a neutrino?". With Dirac neutrinos you get a sterile right handed neutrino, which seems impossible as it doubles the count of neutrinos state at the time on nuclearsynthesis of helium in the big bang, breaking the good fit for helium and deuterium abundance. With standard majorana neutrinos, the overtaken neutrino becomes an anti-neutrino violating lepton number by two units. This is a highly bizarre state of affairs. Merely by changing the velocity of the observer, we changed a particle into an antiparticle. This seems at odds with the principle of relativity. A thought experience might run like this, consider a rocket accerelating away from the sun. On board the spaceship is a tank of chlorine gas. At first the chlorine atoms occasionally are hit by neturinos emitted by the sun, changing into argon atoms. But as the spaceship gets faster, eventually it becomes faster than the neutrinos emitted by the sun. It now seems to run into anti-neutrinos from ahead, and these turn the argon back into chlorine.

With our axial force this is no longer possible, the axial charge on a neutrino is conserved it can never turn into an anti-neutrino, instead when overtaken it must flip its spin to keep its helicacy constant. To do this it must emit a spin-1 particle. Thus we have our basic vertex for neutrino interactions, in figure 1. Then angular momentum conservation requires that a neutrino can only emit a real (on-shell) axiphoton when its momentum reverses thus, emission of axiphoton is strongly constrained for relavistic neutrinos.

Looking at our Languagian from section three, we can immediately solve the Euler-Languange equations to derive the equations for our axiphoton:

$$\partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu B_\nu - \partial^\nu (\partial_\mu B^\mu) = j^\nu \quad (48)$$

And for neutrinos

$$[i\hbar\gamma^\mu\partial_\mu + M + q\gamma^\mu\gamma^5 B^\mu]\phi = 0 \quad (49)$$

It immediately follows (from the antisymmetry of  $F^{\mu\nu}$ ) that the axial charge is conserved, writing  $j^\nu = (\rho, J_x, J_y, J_z)$

$$\partial_\mu j^\mu = 0, \quad \text{or} \quad \frac{d\rho}{dt} - \text{Grad}J = 0 \quad (50)$$

In section 2, we left our majorana equation for the neutrino in form which needed the spinors extended to a larger dimension in order to give a valid solution. We had for the left handed component:

$$i\frac{\partial\phi_l}{\partial t} - i\sigma'_j\frac{\partial\phi_l}{\partial x_j} - m_l R\phi_l = 0. \quad (51)$$

With  $R = R^{-1}$  and  $R\sigma'_i = -\sigma'_i R$ .

Let us extended  $\phi_l$  to a doublet of spinors, then choose the extensions of  $\sigma'$  and representation of  $R$  below

$$\phi_l = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix}, \quad \sigma'_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (52)$$

Where  $\sigma_i$  are the standard Pauli matrices. Then we obtain the coupled equations:

$$i\frac{\partial\phi_a}{\partial t} - i\sigma_j\frac{\partial\phi_b}{\partial x_j} + im_l\phi_b = 0, \quad (53)$$

$$i\frac{\partial\phi_b}{\partial t} - i\sigma_j\frac{\partial\phi_a}{\partial x_j} - im_l\phi_a = 0. \quad (54)$$

Then taking  $\phi_a^\dagger(53) - (53)^\dagger\phi_a + \phi_b^\dagger(54) - (54)^\dagger\phi_b$  we find the mass terms cancel and using our definitions of axial charge density and current from section 3.

$$\rho = -q\bar{\phi}\gamma^5\gamma^0\phi = q(\phi_l^\dagger\phi_l - \phi_r^\dagger\phi_r) \quad (55)$$

and a current in the  $i^{th}$ -direction,

$$J_i = -q\bar{\phi}\gamma^5\gamma^i\phi = -q(\phi_l^\dagger\sigma'_i\phi_l + \phi_r^\dagger\sigma'_i\phi_r) \quad (56)$$

we find the above equations reproduce our continuity equation as required. We find no other choices of 4 by 4 matrices that will lead to a correct continuity equation but change our other results. For details of the possible solutions look at appendix A. Each produces an identical current, force and transition matrix elements.

We need to know what effect the axial force has on our neutrino states, the calculation is analogous to deriving the Lorentz force from the Dirac

equation. The derivation for the axial force starts the same way with the proper time derivative of the momentum [20]

$$\frac{dp^\mu}{d\tau} = -i[H, p^\mu], \quad H = \gamma_\mu \pi^\mu \quad (57)$$

Where  $\pi^\mu$  is the canonical momentum,  $\pi^\mu = p^\mu - qA^\mu$  in the E-m case, and  $\pi^\mu = p^\mu - q\gamma^5 B^\mu$  for the axial case. Then

$$\frac{dp^\mu}{d\tau} = [\partial_\mu, \gamma_\nu \gamma^5 B^\nu] = \gamma^5 \gamma_\mu \left[ \frac{\partial B^\nu}{\partial x^\mu} - B^\nu \partial^\mu \right] \quad (58)$$

Now the term with the trialing derivative is a problem, but starting from the Dirac equation and its adjoint,

$$(\gamma^\mu \pi_\mu - M)\phi = 0 \quad , \quad \bar{\phi}(\gamma^\mu \pi_\mu - M^\dagger) = 0, \quad (59)$$

multipling the two and reusing the dirac equations to remove the terms with a signal mass matrix gives:

$$\bar{\phi}(\gamma^\mu \gamma^\nu [\partial_\mu \partial_\nu + q^2 B_\mu B_\nu - i\partial_\mu B_\nu - iB_\mu \partial_\nu])\phi + \bar{\phi}M^\dagger M\phi = 0. \quad (60)$$

Above the real and imaginary parts must be equal to zero seperately, and using the anticommutation rules for the gamma matrixes

$$\gamma^\mu \gamma^\mu = 2 \quad \Rightarrow \quad \langle \phi | B_\mu \partial_\mu | \phi \rangle = - \langle \phi | \partial_\mu B_\mu | \phi \rangle, \quad (61)$$

$$\gamma^\mu \gamma^\nu = -\gamma^\nu \gamma^\mu \quad \Rightarrow \quad \langle \phi | B_\mu \partial_\nu | \phi \rangle = \langle \phi | \partial_\nu B_\mu | \phi \rangle, \quad \mu \neq \nu. \quad (62)$$

Thus the expectation values of the force on a particle are:

$$\langle \phi | F^\mu | \phi \rangle = \langle \phi | \frac{dp^\mu}{d\tau} | \phi \rangle = \langle \phi | \gamma^\nu \gamma^5 \left[ \frac{\partial B^\nu}{\partial x_\mu} - \frac{\partial B^\nu}{\partial x^\mu} \right] | \phi \rangle = \langle \phi | \gamma^\nu \gamma^5 G_{\mu\nu} | \phi \rangle \quad (63)$$

Now if the field strength  $G_{\mu\nu}$  is approximately unchanging over the positions with significant values of the wavefunction, we can separate it out of the matrix element to get:

$$\langle \phi | F^\mu | \phi \rangle = \langle \phi | \gamma^\nu \gamma^5 | \phi \rangle G_{\mu\nu} \quad (64)$$

This is the same equation as for electrodynamics apart from the  $\gamma^5$  chiral operator, and for eigenstates of the Dirac equation, the Lorentz force is quickly recovered. For the axial force we can return to our definition the current (20). So find:

$$\langle \phi | F^\mu | \phi \rangle = j^\nu G_{\mu\nu} \quad (65)$$

We do not however yet, have a explicit form of the current. To find this we will need find solutions of equations (53) and (54). This isn't too differicult

we proceed in the standard way used for the Dirac equation, and look for plain wave solutions of different energy  $E$  and momentum  $p$ . Then we have

$$\phi_a = u_a(e, \mathbf{p})e^{i(-\omega t + \mathbf{k} \cdot \mathbf{x})} \quad , \quad \phi_b = u_b(e, \mathbf{p})e^{i(-\omega t + \mathbf{k} \cdot \mathbf{x})} \quad (66)$$

With  $E = 2\pi\hbar\omega$ ,  $\mathbf{p} = 2\pi\hbar\mathbf{k}$  as usual. Then

$$(E/c)u_b - (\mathbf{p} \cdot \vec{\sigma} + imc)u_a = 0 \quad , \quad (E/c)u_a - (\mathbf{p} \cdot \vec{\sigma} - imc)u_b = 0 \quad (67)$$

or,

$$u_b = \frac{c(\mathbf{p} \cdot \vec{\sigma}) + imc^2}{E}u_a \quad , \quad u_a = \frac{c(\mathbf{p} \cdot \vec{\sigma}) - imc^2}{E}u_b \quad (68)$$

We may choose four states, the first two with  $u_a = (1, 0)^T$  or  $(0, 1)^T$ , and the second two with  $u_b = (1, 0)^T$  or  $(0, 1)^T$  with  $c = 1$ :

$$u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ (-im + p_z)/E \\ (im + p_x - ip_y)/E \end{pmatrix}, \quad u_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ (-im + p_x + ip_y)/E \\ (im - p_z)/E \end{pmatrix}, \quad (69)$$

$$u_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} (p_z + im)/E \\ (p_x + ip_y + im)/E \\ 1 \\ 0 \end{pmatrix}, \quad u_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} (p_x - ip_y + im)/E \\ (-p_z + im)/E \\ 0 \\ 1 \end{pmatrix} \quad (70)$$

We may form a linear combination of  $u^{(1)}$  and  $u^{(2)}$  and similar of  $u^{(3)}$  and  $u^{(4)}$

$$v_1 = a.u_1 + b.u_2 \quad , \quad a^*a + b^*b = 1, \quad (71)$$

Then form a spinor by letting  $s = (a, b)$

$$v_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} s \\ \mathbf{p} \cdot \vec{\sigma}s/E - ims/E \end{pmatrix} \quad , \quad v_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{p} \cdot \vec{\sigma}s/E + ims/E \\ s \end{pmatrix} \quad , \quad (72)$$

The second state above, however is not orthogonal to the first,

$$v_1^\dagger \cdot v_2 = s^\dagger(\vec{\sigma} \cdot \mathbf{p})s, \quad (73)$$

but using the Gram Schmidt procedure we can form a state that is. First let  $\mathbf{s} = \mathbf{s}^\dagger \sigma \mathbf{s}$  then we may write the above as a dot product:  $s^\dagger(\vec{\sigma} \cdot \mathbf{p})s = (\mathbf{s} \cdot \mathbf{p})$ . The state  $u_4$  is still orthogonal to  $u_1$ , as are  $u_2$  and  $u_3$ , so we may form a state orthogonal to  $v_1$  as

$$w = v_2 - (v_1^\dagger \cdot v_2)v_1 = \frac{1}{\sqrt{2}\sqrt{E^2 - (\mathbf{s} \cdot \mathbf{p})^2}} \begin{pmatrix} [\vec{\sigma} \cdot \mathbf{p} + im - (\mathbf{s} \cdot \mathbf{p})]s \\ E + (\mathbf{s} \cdot \mathbf{p})[im - (\vec{\sigma} \cdot \mathbf{p})] \end{pmatrix}. \quad (74)$$

So we have four independent states, this looks like two much for a neutrino, which is supposed to have only one spin state. But lets compute the helicity of one of these states.

$$h = \langle v | \frac{\mathbf{p}}{2|\mathbf{p}|} \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} | v \rangle = \frac{1}{2E|p|} s^\dagger (\vec{\sigma} \cdot \mathbf{p})^2 s = \frac{p^2}{2E|p|} \quad (75)$$

Something wonderful happens, the mass terms cancel out, and we are left with the same value of helicity for each of the states. The only way it seems possible to reverse the helicity of the particle is to take a negative energy solution. Thus all the neutrinos have the same helicity and all anti-neutrinos have the opposite helicity. Exactly as required for a neutrino wavefunction.

Returning to our derivation of the axial analog to the EM lorenz force we, can now find the charge and current due to our wavefunction,  $v_1$

$$\rho = qv^\dagger v = q \quad (76)$$

$$J_i = \frac{q}{2} s^\dagger \frac{[\sigma_i \vec{\sigma} \cdot \mathbf{p} + \vec{\sigma} \cdot \mathbf{p} \sigma_i]}{E} s = q \frac{(\mathbf{s} \cdot \hat{\mathbf{i}})(\mathbf{p} \cdot \hat{\mathbf{i}}) \hat{\mathbf{i}} + (\mathbf{s} \cdot \hat{\mathbf{j}})(\mathbf{p} \cdot \hat{\mathbf{j}}) \hat{\mathbf{j}} + (\mathbf{s} \cdot \hat{\mathbf{k}})(\mathbf{p} \cdot \hat{\mathbf{k}}) \hat{\mathbf{k}}}{E} \quad (77)$$

Where  $\mathbf{s} = s^\dagger \vec{\sigma} s$ . The current has a unusual form with a product that looks like a vector valued version of the dot product. This would not normally be valid but the spin and momentum are in different spaces so can transform differently to preserve the observable current. Lets use the symbol  $\odot$  for the above product, then:

$$j = q \frac{\mathbf{p} \odot \mathbf{s}}{E} = q \mathbf{v} \odot \mathbf{s} \quad (78)$$

Using the second wavefunction gives a more complicated result.

$$j(v_2) = q \frac{\mathbf{p} \odot \mathbf{s}}{E} - 2 \frac{\mathbf{p}}{E} \frac{(\mathbf{s} \cdot \mathbf{p})^2}{(E^2 - \mathbf{s} \cdot \mathbf{p})^2} \quad (79)$$

Where,  $\beta = |v|/c$  and  $\gamma = 1/\sqrt{1 - \beta^2}$ . Taking a general wavefunction,

$$\phi = Av_1(s) + Bw(s) \quad (80)$$

Gives the current as:

$$J = \mathbf{s} \odot \mathbf{v} - 2B \frac{\mathbf{p}}{E} \frac{(\mathbf{s} \cdot \mathbf{p})^2}{(E^2 - \mathbf{s} \cdot \mathbf{p})^2} + \frac{Re(A^\dagger B)}{E \sqrt{(E^2 - (\mathbf{s} \cdot \mathbf{p})^2)}} [2(\mathbf{s} \cdot \mathbf{p})(\mathbf{s} \odot \mathbf{p}) - \mathbf{p}(\mathbf{s} \cdot \mathbf{p})] \quad (81)$$

Under relativistic transformations, charge should transform to current as,  $J_z = \gamma(J'_z + v\rho')$  this seems to demand that  $\mathbf{s} = \hat{\mathbf{p}}$  giving only a single neutrino spin state. It is easy to see that the above current will always be smaller than  $qv$  for all  $B \neq 0$ . Thus taking a standard current proportional to the velocity of the particle, enforces that there is only one possible wavefunction representing the particle.

We indeed require there be only one state for a neutrino, if it were otherwise, neutrinos would create too much radiation pressure in the early big bang. At this stage however we are not certain of the exact way the other states are forced out of existence. E.g. whether they never exist, or whether they are only cancelled out in the classical limit. It is an interesting problem for which more work may need to be done. If we do take the spin in the momentum direction, then our force and current equations immediately reduce to the standard ones for electromagnetism. But how can it be true that the spin is in the momentum direction in all frames? Perhaps it need not be true. A simple simulation of a neutrino moving in a random axi-magnetic field that is random around  $B = 0$  shows that the spin turns to point in the momentum direction. A plasma of neutrinos, like any plasma should about a magnetic field, and move to expel it, thus a group of neutrinos should act collectively to form a frame with  $B = 0$ , and align their spins in their direction of motion in that frame. It requires further mathematic proof that this will always occur, but that is beyond the scope of this work.

With the standard current the neutrino obeys the same Lorentz force law as for electromagnetic field. I.e.

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \quad (82)$$

Where  $\mathbf{E}$  is the axi-electric field and  $\mathbf{B}$  is the axi-magnetic field.

Like the electron, the neutrino should also have a magnetic moment, or axi-magnetic moment in this case. We can derive it, following the text book [22] derivation. Starting with the neutrino wave equations and adding minimal coupling,  $E \rightarrow E - q\gamma^5\phi$ ,  $p^\mu \rightarrow \pi^\mu = p^\mu - q\gamma^5 A^\mu$ .

$$(E - q\gamma^5\phi)u_b/c - (\pi \cdot \sigma - imc)u_a = 0, \quad (E - q\phi)u_a/c - (\pi \cdot \sigma + imc)u_b = 0 \quad (83)$$

Then,

$$(E - q\gamma^5\phi)^2 = c^2(\pi \cdot \sigma)^2 + m^2c^4 = c^2(\mathbf{p} - q\gamma^5\mathbf{A}^\mu)^2 + i\gamma^5\sigma(\pi \times \mathbf{A} + \mathbf{A} \times \pi)c^2 + m^2c^4 \quad (84)$$

Let  $E = mc^2 + W$ , then after  $p \times A + A \times p = -i\hbar B$ :

$$(W + mc^2 - q\gamma^5\phi)^2 = c^2(p - q\gamma^5 A)^2 + c^2\hbar q\gamma^5 B + m^2c^4 \quad (85)$$

In the non-relativistic limit, the mass term dominates and we can ignore the second order of small quantities on the left. Leading to:

$$W = \frac{1}{2m}(p - q\gamma^5 A)^2 + q\phi + \frac{q\hbar}{2m}\gamma^5\sigma \cdot B \quad (86)$$

Thus the neutrino axi-magnetic moment is,

$$g = \frac{q\hbar}{2m} \quad (87)$$

And the energy eigenvalues are, ignoring second order in velocity,

$$E_+ = g\mathbf{s} \cdot \mathbf{B} + g\frac{\mathbf{s} \cdot \mathbf{v} \times \mathbf{B}}{c}, \quad E_- = g\mathbf{s} \cdot \mathbf{B} - g\frac{\mathbf{s} \cdot \mathbf{v} \times \mathbf{B}}{c} \quad (88)$$

$E_+$  corresponds to choosing the  $v^{(1)}$  solution, and  $E_-$  corresponds to choosing the  $v^{(2)}$  solution.

## 11 Lorentz Transformations of the wave equation

In the last section we came up with a current that doesn't seem to transform correctly under the relativistic boosts. Why is this and indeed how do our spinor wavefunctions transform under boosts? Lets start again with our left handed wave equation (51), this looks like a massive Weyl equation which doesn't exist in 4-dimensions. Indeed we had to extend each chiral half of the wavefunction to four complex components to get a solution. Let us take the momentum space version of the equation and see how it transforms under a boost.

$$(E - \vec{\sigma} \cdot \mathbf{p})\phi = Rm\phi \quad (89)$$

Under a boost, the energy and momentum transform as:

$$E' = \gamma E + \gamma \vec{\beta} \cdot \mathbf{p} \quad , \quad \mathbf{p}' = \gamma \vec{\beta} E + (\gamma - 1)(\mathbf{p} \cdot \vec{\beta})\hat{\beta} + \mathbf{p} \quad (90)$$

Where  $\vec{\beta}$  is velocity vector divided by the speed of light. While left handed spinors transform as:

$$\phi' = \frac{1}{\sqrt{2}} \left[ \sqrt{\gamma + 1} + \vec{\sigma} \cdot \hat{\beta} \sqrt{\gamma - 1} \right] \phi \quad (91)$$

Then starting with the wave equation in the boosted frame:

$$(E' - \vec{\sigma} \cdot \mathbf{p}')\phi' = Rm\phi', \quad (92)$$

and using the properties of the  $\gamma$  and  $\beta$  functions,

$$\sqrt{(\gamma - 1)(\gamma + 1)} = \gamma\beta, \quad (93)$$

$$\gamma\beta\sqrt{\gamma + 1} - \gamma\sqrt{\gamma - 1} = \sqrt{\gamma - 1}, \quad (94)$$

$$\gamma\sqrt{\gamma + 1} - \gamma\beta\sqrt{\gamma - 1} = \sqrt{\gamma + 1}, \quad (95)$$

we find the left handed to spinor transformation to a right handed one ending up with

$$\begin{aligned} & \left[ \sqrt{\gamma + 1} - \vec{\sigma} \cdot \hat{\beta} \sqrt{\gamma - 1} \right] (E - \vec{\sigma} \cdot \mathbf{p}) \phi \\ &= Rm \left[ \sqrt{\gamma + 1} + \vec{\sigma} \cdot \hat{\beta} \sqrt{\gamma - 1} \right] \phi \end{aligned} \quad (96)$$

So provided  $R\sigma = -\sigma R$  the standard transformation works just fine. Which is the case with our extended  $\sigma'$ .

We could also use  $\sigma_3 \otimes (\vec{\beta} \cdot \vec{\sigma})$  as the symbol in the boost transformation. But this would lead to a very different transformation of the current under boosts.

## 12 Neutrino interactions calculated

In order to do calculation with our the axial force, we need to find the Feynman rules for particles propagators and vertices. Lets start by finding the propagator for the neutrino. From our Langragian when we remove the interaction parts, to get:

$$\mathcal{L}_0 = i\bar{\phi}(\gamma^i\partial_i - \bar{\phi}M)\phi = \bar{\phi}S^{-1}\phi \quad (97)$$

We may then find the Propagator  $iS$  by inverting the term between the wavefunctions [23].

$$S(x) = (i\gamma.\partial + M^\dagger)\Delta_F(x) \quad (98)$$

Where the Feynman propagator  $\Delta_F$  is defined by

$$(\square + M^2)\Delta_F(x - y) = -\delta^4(x - y) \quad (99)$$

Our inversion works correctly due to the properties of our extended Dirac algebra. In particlar we required:

$$\gamma^i M^\dagger - M^\dagger \gamma^i = 0, \quad MM^\dagger = \text{diag}(m_l^2, m_r^2), \quad \bar{\phi}(\gamma.\partial)^2\phi = \phi_l^\dagger \square \phi_l + \phi_r^\dagger \square \phi_r \quad (100)$$

Which can be easierly varified for our choice of mass matrix and our extension of the Dirac matrices to 8 by 8.

We then find that we can the propagator seperates into two seperate propagators for left and right handed neutrinos, with no conversion from left to right handed states. In momentum space,  $i\hbar\partial_i = p_i$ . The propagator for both states is:

$$\bar{u}_l S u = u^\dagger \gamma^0 \frac{\gamma \cdot p + M^\dagger}{(2\pi)^4 (p^2 - M^\dagger M)} u \quad (101)$$

So using  $u_l^\dagger$  and  $u_r^\dagger$  is the output states rather than  $\bar{u}$ , we have:

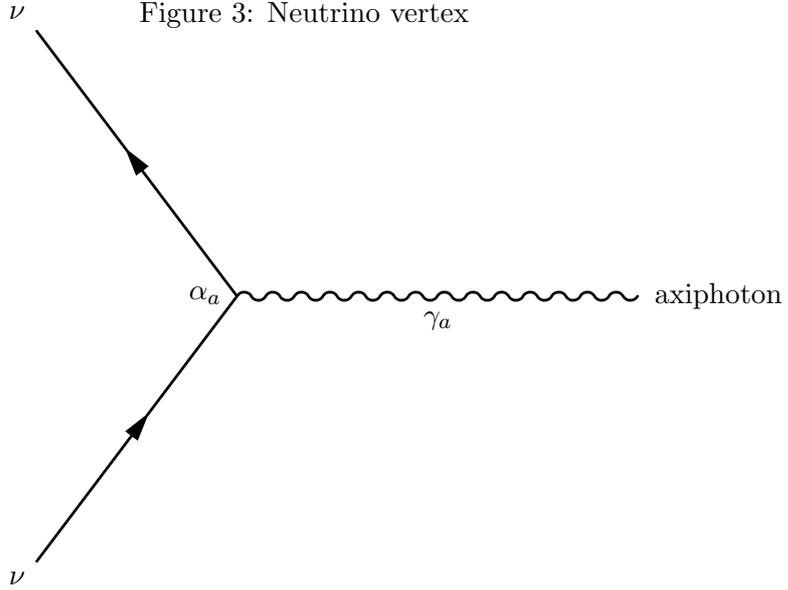
$$S_l = \frac{1}{(2\pi)^4} \frac{I_4 E - \sigma_1 \otimes \sigma_i p_i + R m_l}{p^2 - m_l^2}, \quad S_r = \frac{1}{(2\pi)^4} \frac{I_4 E + \sigma_1 \otimes \sigma_i p_i + R m_r}{p^2 - m_l^2} \quad (102)$$

The  $R$  matrix could be written as  $\sigma_2 \otimes I_2$  but we think it is clearer to use the Pauli matrices as symbols only for the inner components. The axiphoton field is a guage field generated in the same way as the photon field and so the axi-photon propagator should be identical to the photon propagator in which ever guage we choose to use.

$$P_\gamma = \frac{-ig_{\mu\nu}}{q^2} \quad (103)$$

Finally the axial force has just the one vertex, see figure 1.

$$V_a = ig_a \gamma^5 \gamma^\mu, \quad g_a = q\sqrt{4\pi\alpha_a} \quad (104)$$



### 12.1 The decay of right hand neutrinos

We are now in a position to calculate neutrino interactions. By far the simplest is the decay of the right handed neutrino, into an axiphoton and an antineutrino. This is described just by a single vertex, as in figure 1. We put the anti-(right handed neutrino) (so a left handed state) in the rest frame, and take the approximation that the antineutrino is massless. Then the kinematics is given the conversion of 4-momentum. The neutrino and the axiphoton will be emitted back to back, lets say in the z-direction.

$$(m_r c, 0, 0, 0) = (\sqrt{m_l^2 c^2 + p^2}, 0, 0, p) + (p, 0, 0, -p) \quad (105)$$

Then

$$|p| = \frac{m_r^2 - m_l^2}{2m_r} c \approx \frac{1}{2} m_r c \quad (106)$$

The amplitude for the process is then from the standard Feynman rules e.g [24]

$$M = u^\dagger i g_a \gamma^5 \gamma^\mu \epsilon^{*\mu} \nu \quad (107)$$

If our axiphoton is a real particle it must have transverse polarisation so  $\epsilon^\mu = (0, a, b, 0)$  where  $a^2 + b^2 = 1$  and both  $a$  and  $b$  are real. Our incoming right handed neutrino  $\bar{\nu}$  has zero momentum, While our outgoing neutrino  $u$  has approximately zero mass. It is normal to normalise the wavefunction

to  $u^\dagger u = 2|E|/c$ . So we put

$$u = \sqrt{\frac{|E|}{c}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0_4 \end{pmatrix}, \quad v = \sqrt{\frac{|E|}{c}} \begin{pmatrix} 0_2 \\ 0_2 \\ s \\ is \end{pmatrix} \quad (108)$$

With our choice of dirac matrices  $C = iR[\ ]^*$  and  $P = \gamma^0$  so that  $u^\dagger = -(is, s, 0, 0)$ . Also  $\gamma^0 \gamma^5 \gamma^\mu = -I_4 \sigma^\mu$  for  $\mu = 1, 2$  or  $3$ .

Because the helicity of the neutrino reverses, (which is the whole point of the exercise), we also pick up an additional factor,  $2(m_l/m_r)^2$

Thus,

$$M = 2ig_a \left(\frac{m_l}{m_r}\right)^2 \frac{E}{c} (is + s) [\sigma_1 a + \sigma_2 b] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (109)$$

We see that the input anti-RH neutrino must have left circular spin, then the output neutrino is right circular, thus the axi-photon must be left circular:  $(a, b) = (1, -i)/\sqrt{2}$ . Then  $|M|^2 = 2E^2 \left(\frac{m_r}{m_l}\right)^2 g_a^2 |(0, 1) \cdot s|^2 / c^2$ . The lifetime is then [24]

$$\Gamma = \frac{m_l^2}{(4\pi)^2 \hbar m_r^3} \int \frac{|M|^2}{|\mathbf{p}|^2} \delta(m_r c - 2|\mathbf{p}|) d^3 \mathbf{p} = \frac{g_a^2 m_l^2 c^2}{4(4\pi)^2 \hbar m_r} \int [(0, 1) \cdot s]^2 \sin \theta d\theta d\phi \quad (110)$$

It is entirely equivalent to integrate over the initial spin state of the neutrino, as it is to integrate over the output momentum. So the integral is

$$I = \int \left| \begin{pmatrix} 1 & 0 \\ \cos \theta & \sin \theta [\cos \phi - i \sin \phi] \\ \sin \theta [\cos \phi + i \sin \phi] & \cos \theta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \sin \theta d\theta d\phi \quad (111)$$

Then  $I = 8\pi/3$ . So that the total decay rate is

$$\Gamma = \frac{g_a m_l^2 c^2}{6\hbar m_r} \quad (112)$$

We see that right handed neutrinos decay quickly if there are light, which protects them from being detected in many experiments. This is good since our theory predicts that they will be created at a branching fraction of around  $1/\alpha_a$  in pion decays. Comparing our decay rate with the rate Schevetz et al. needed to explain the LSND anomaly [16] by sterile neutrino decay we have  $m_4 = 50 - 750 eV$  just in the right range to save matter from neutrino Fermi pressure.

## 12.2 Neutrino-Neutrino scattering

An important limit to any forces experienced by neutrinos is that they do not lead to a too large a scattering cross section at high energy. Neutrinos

made it safely to earth from the SN1987A supernova across some hundred thousand light years, while doing so they passed by the cosmic neutrino background left over from the big bang which is at least 300 neutrinos per  $cm^3$ . Thus [25] the neutrino-neutrino cross section must be smaller than  $10^{-25}cm^2$  for the supernova neutrinos at  $E = 1 - 15MeV$ . Lets calculate how large a cross section the axial force gives neutrinos to interact.

First lets calculate the neutrino equivalent to Mott scattering. With two different types of neutrino colliding the only interaction at tree level is the S-channel, whose amplitude is.

$$M = -\frac{g_a^2}{(p_1 - p_3)^2} [u^\dagger(3)\Gamma_\mu u(1)] g^{\mu\nu} [u^\dagger(4)\Gamma_\nu u(2)] \quad (113)$$

The first particle with mass  $m$  enters as  $u(1)$  and leaves as  $u(3)$ , while the second with mass  $M$  enters as  $u(2)$  and leaves as  $u(4)$ . Will keep the  $u$ 's normalised to 1.  $\Gamma_\mu$  is the left handed part of  $\gamma^5\gamma_\mu$ . And so is,

$$\Gamma_\mu = \begin{cases} I_4, & \mu = 0 \\ \sigma'^\mu, & \mu = 1, 2, 3 \end{cases} \quad (114)$$

The right handed version would pick up a minus sign on the  $\sigma'$ s but overall would lead to the same result.

Casimir's track for finding the square amplitude when we sum/average over the spins, still works in the neutrino case, and the completeness relation for the neutrino spins is.

$$\sum_{s=1..4} u_s u_s^\dagger = I + \frac{\sigma' \cdot pc + mc^2 R}{E} \quad (115)$$

Then averaging over incoming particles and summing over outgoing leads to:

$$\begin{aligned} \langle |M|^2 \rangle &= \frac{g_a^4}{16E_1 E_2 E_3 E_4 (p_1 - p_3)^4} \\ &\times \text{Tr} [\Gamma_\mu (E_3 + \sigma' \cdot p_3 + mR) \Gamma_\alpha (E_1 + \sigma' \cdot p_1 + mR)] g^{\mu\nu} g^{\alpha\beta} \\ &\times \text{Tr} [\Gamma_\nu (E_4 + \sigma' \cdot p_4 + mR) \Gamma_\beta (E_2 + \sigma' \cdot p_2 + mR)] \end{aligned} \quad (116)$$

The traces evaluate quite easily, since an odd number of  $\sigma$ s or  $R$ s leads to a traceless value. The times the matrices make a difference are  $\text{Tr}(\sigma R \sigma R) = -1$  and  $\text{Tr}[\sigma_i(\sigma \cdot \mathbf{p})\sigma_i(\sigma \cdot \mathbf{q})] = 2(p \cdot \hat{e}_i)(q \cdot \hat{e}_i) - p \cdot q$ . Where  $\hat{e}_i$  is a the  $i^{th}$  basis vector,  $\hat{e}_1 = \hat{i}$ ,  $\hat{e}_2 = \hat{j}$ ,  $\hat{e}_3 = \hat{k}$ . Naming the terms in the traces as  $T^{\mu\nu}$

$$T_{\mu\nu} = \text{Tr} [\Gamma_\mu (E_3 + \sigma' \cdot p_3 + mR) \Gamma_\alpha (E_1 + \sigma' \cdot p_1 + mR)]$$

We find the traces are:

$$T^{\mu\nu} = \begin{cases} 4[E_1 E_3 + p_1 \cdot p_3 + m^2] & \text{for } \mu = \nu = 0 \\ 4[E_1 E_3 - p_1 \cdot p_3 - m^2 + 2(p_1 \cdot \hat{e}_\mu)(p_2 \cdot \hat{e}_\mu)] & \text{for } \mu = \nu = 1, 2 \text{ or } 3 \\ 4(p_1 \cdot \hat{e}_\mu)(p_3 \cdot \hat{e}_\nu) + 4(p_1 \cdot \hat{e}_\nu)(p_3 \cdot \hat{e}_\mu) & \mu \neq \nu, \mu \neq 0, \nu \neq 0 \\ 4E_1(p_3 \cdot \hat{e}_\mu) + 4E_3(p_1 \cdot \hat{e}_\mu) & \mu \neq 0, \nu = 0 \\ 4E_1(p_3 \cdot \hat{e}_\nu) + 4E_3(p_1 \cdot \hat{e}_\nu) & \nu \neq 0, \mu = 0 \end{cases} \quad (117)$$

And similarly for the second trace.

Then

$$\langle |M|^2 \rangle = T_{\mu\alpha} g^{\mu\nu} g^{\alpha\beta} T_{\nu\beta} \quad (118)$$

If go to four momentum vector,  $k_1 = (E_1, p_1)$ , we find the scattering matrix element finally evaluates to

$$\langle |M|^2 \rangle = \frac{2g_a^4 [(k_1 \cdot k_2)(k_3 \cdot k_4) + (k_1 \cdot k_4)(k_2 \cdot k_3) + m^2(k_2 \cdot k_4) + M^2(k_1 \cdot k_3) + 2m^2 M^2]}{E_1 E_2 E_3 E_4 (k_1 - k_3)^4} \quad (119)$$

Where we have the 4-vector scalar product as having a negative energy part  $k_1 \cdot k_2 = p_1 \cdot p_2 - E_1 E_2$ .

It is clear that the scattering is strongly suppressed by the neutrino energy and so should allow fast moving neutrinos to evade interactions via the axial force.

### 12.3 A renormalisable theory?

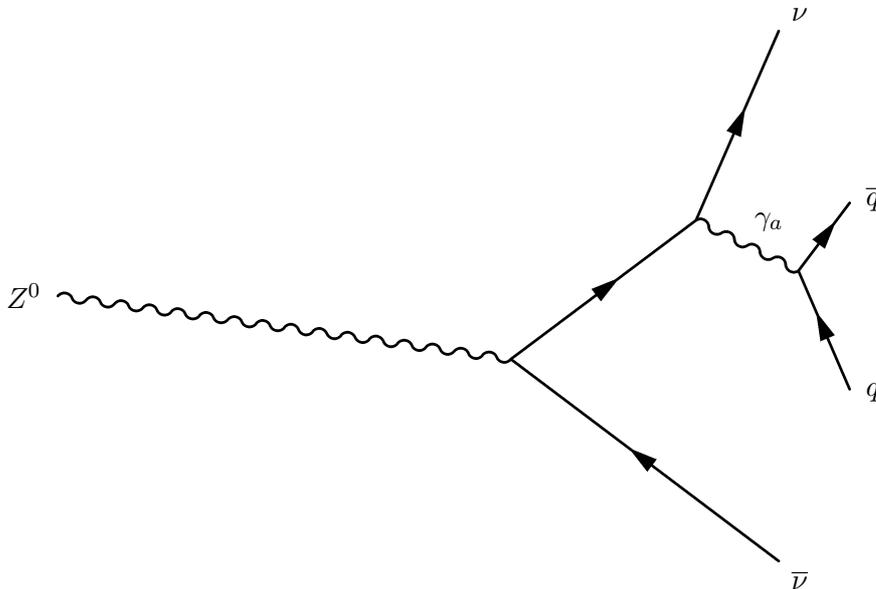
Its not too difficult to invent a Feynman vertex and claim a new theory. Actually showing that the theory is renormalisable and gives consistant results is another matter, and we do not claim to have proof of this. Because of its similarity to QED (both being U(1) theories) many of the QED results should also hold for our axial theory (QAD?). In particular the Ward Identities look identical, and the triangle diagrams should also cancel out, due to the there being antiparticles, and due to the freedom to reverse the direction of the internal momenta. Some tricks used to renormalise QED won't work here, however, in particular dimensional regularisation [23] (pg 344), won't work, since  $\gamma^5$  is only defined in exactly 4 dimensions. So we are left with plenty of work to proof that QAD is a physical theory. We will however not attempt to do this here.

## 13 Connection with Standard Model Experiments

### 13.1 Invisible Z decays, The neutrino count anomaly

The number of standard model left handed neutrinos can easierly be measured, by the measuring the number events in which a Z boson decays to purely invisible end products. The result is suprisingly slightly smaller than

Figure 4: Solving the neutrino count anomaly, an invisible  $2\nu$  decay mode of the  $Z^0$  becomes a visible hadronic decay mode.



3. Measurement show it to be  $2.984 \pm 0.008$  [1]. Our model in fact predicts that measurement is reduced by a factor of about  $\alpha_a(Z)$ . This is due to the fact the neutrinos may emitted axi-photons which then pair produces quarks resulting in a visible hadronic decay. If all the deficit in neutrinos is due to axiphoton production we require that  $\alpha_a$  has grown to around  $1/375$  at the  $Z$  mass of  $\approx 90\text{GeV}/c^2$ . This is a little high, but we think a smaller value could account for some missing invisible width, without upsetting other measurements.

### 13.2 Precision electroweak data

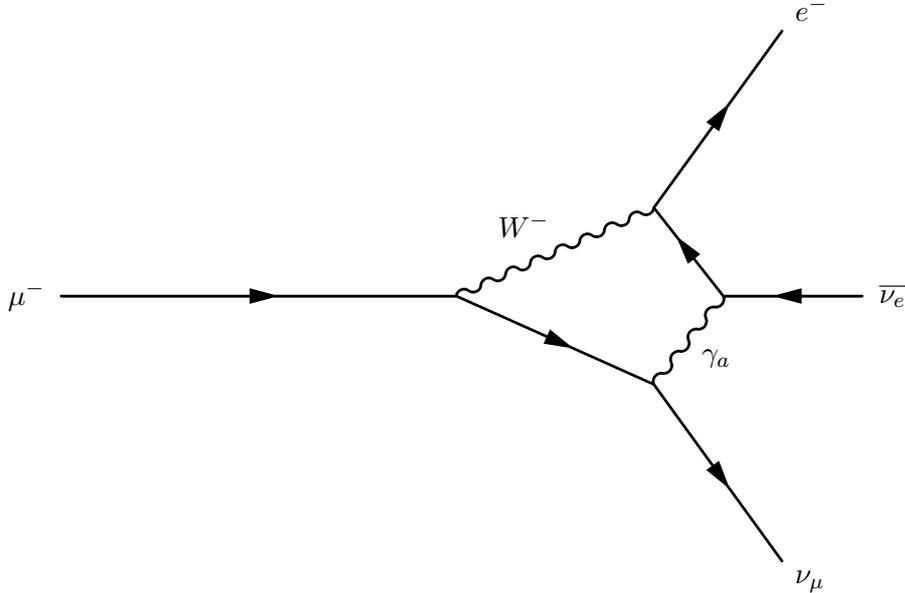
We need to show that our axial force is compatible with precision electroweak data. Since we expect a force constant of around  $\alpha/60$ , any correction will be small, however electroweak data is getting reasonably precise. The weak force constant, is most accurately derived from the muon decay constant, however this will pick up a small correction due to the axial force, due to an possible interaction between the outgoing neutrinos, see our diagram.

Thus  $G_F$  the fermi constant, will get smaller by a factor.

$$G_{Fa} = G_F \left( 1 - \frac{1}{\alpha_a} \right)$$

Most of the other weak processes do not have two outgoing neutrinos and so will be smaller by this same factor. Current electroweak measurements thus

Figure 5: The axial force increases the muon decay rate for a given weak force strength due to this diagram.



do not seem yet to have the precision great enough to detect the change given above, and so our theory seems to be compatible with current electroweak data. It work be interesting to see if any our the outstanding experimental variances with standard electroweak model can be solved by incorporating the axial force. One such variance is the as the NuTeV anomaly [3] found in the electroweak interaction by the NuTeV collaboration at fermilab [2]. Another possiblity is the forward backward asymmetry in  $B$  meason decays. It would be interesting to see if these could be solved by the axial force.

The results LSND experiment seems to be fit by the decay of a right handed neutrino [16]. Miniboone seems to have ruled out an explation of the LSND result by neutrino oscillations, but shows a definite excess of electron neutrinos at the lowest energies. This is very suggestive of a rapid neutrino decay. However with only 3 data points at low energy we did not have enough data to try to fit for a neutrino mass and decay constant.

## 14 Astrophysical Mysteries Solved

### 14.1 The solar corona

One of the closest unsolved mysteries in astrophysics is that of the solar corona [43]. The surface of the sun, the photosphere has a temperature of about 6000K, moving outwounds into the solar atmosphere, the chromosphere the temperature remains similar, then at around 1500Km up, the

temperature suddenly jumps in a matter of 100Km to over a million degrees. It is thermodynamically impossible for this to happen without an external energy source, but what is it that provides the this energy? Explanation such as sound waves, Aifken wave [41] (low frequency plasma oscillations), and the reconnection of the magnetic fields [42] have been given. But none have satisfactory explained the mystery to date. Using modified particle physics one explanation [44] is that the energy is supplied by the decay of solar axions trapped in orbit around the sun. This is better, but still does not quite seem to explain why the transition is quite so sudden.

We propose another decaying particle explanation, this time due to a decaying right handed neutrino. Looking back to our earlier section on the Fermi energy of neutrinos, we find that at high enough density right handed neutrinos are required to stabilise matter against the repulsive effect of the axial force. As the pressure reduces they are no longer needed and can now decay. If they have a finite lifetime then can travel a distance before the decay occurs. And when they do decay, they will supply energy and pressure further into the higher regions of the solar atomosphere. The pressure effect will push back on solar chromosphere. This results in a rapid change in both density and temperature, just above the height at which RHN neutrino decay becomes possible. We can back this up with some a little quantitative analysis resulting in a mass and lifetime for the lightest right handed neutrino. At the top of the corona, the density is:

$$\rho = 10^{-10} Kg/m^3 = 10^{-10} g/cm^3 \quad (120)$$

Using Avagrados constant, and with axial charges of 1/2 on a proton and 1 on a neutrino, we get a neutrino number density of

$$N_\nu = 3 \times 10^{13} cm^{-3} \quad (121)$$

This then gives an Fermi energy for the heaviest left handed neutrino of:

$$E_f = \frac{1.76}{m_\nu} eV \quad (122)$$

If the mass of the heaviest left handed neutrino is  $60meV$ , then that gives  $E_f = 30eV$ . This gives our prediction for the mass of the lightest right handed neutrino. The actual figure might be a few times higher than this, if the corona transition occurs slightly deeper. This neutrino must then live long enough to travel up to 100Km/ Assuming a velocity near the speed of light, say  $1/2c$  on average. This gives a lifetime of about  $6 * 10^{-6}$  seconds.

How does our calculation of neutrino lifetime compare with lifetime we measured above. Not badly at  $30eV$  for the right handed state, and assuming a decay to an  $1meV$  left hander (our first and simplest guess must that the RH states are in the same mass hierarchy as the LH states), we obtain a

lifetime of  $1.07 * 10^{-5}$  seconds, which is an good fit, and becomes even better if we assume, the neutrinos have a thermal velocity,  $\frac{3}{2}kT = c/60$

This is a double win for our theory not only have we correctly described solar corona transitions, we have accurately calculated the size of the transition region from our purely theoretical calculation of neutrino lifetime.

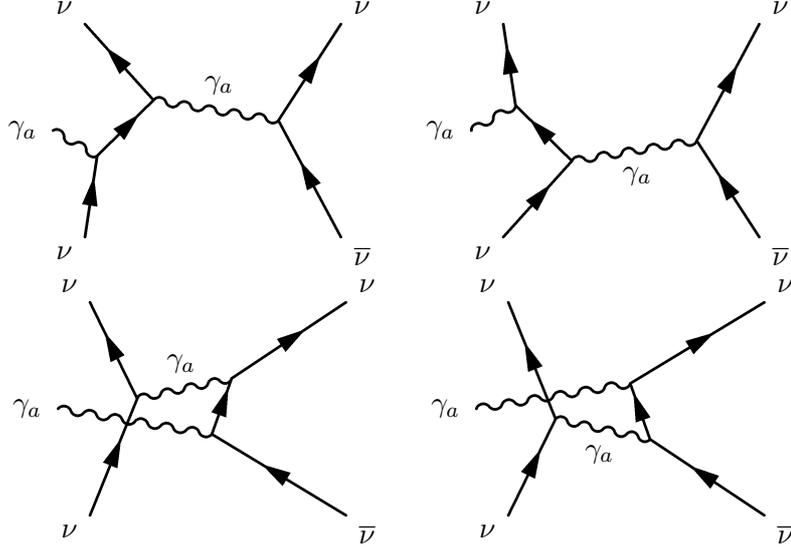
## 14.2 Dark energy

Dark energy is the biggest mystery in cosmology. In 1998 when supernova observations become powerful enough to observe the rate of change of the expansion of the universe, it was found that the universe was not in fact decelerating, as would be expected from effect of gravity, but accelerating. In General Relativity gravity is produced both by energy and by pressure, to cause a repulsive effect, requires some fluid with negative (self attracting) pressure. Such a fluid may be modelled by its equation of state, which gives its pressure in terms of its mass density,  $p = \omega\rho$ . Current observations suggest that  $\omega \approx -1$ . Dark energy is the name for whatever fluid is causing this acceleration of the expansion, and it seems smoothly distributed throughout the universe. Current observations suggest that the universe is composed of about 26.5% matter and 73% dark energy. Any theory of dark energy should explain both the value of  $\omega$ , and also why the amount of dark energy is similar to the amount of matter, which is known as the coincidence problem.

No existing theory has been able to explain all the above data. The simplest model, the cosmological constant, adds a repulsive term by hand to Einsteins gravity theory. Other models include quintessences which adds an extremely low mass  $10^{-33}eV$  scalar field attracted by a very flat potential. Neither model is well justified by particle physics. The fact that the density of dark energy  $\approx (2meV)^4$  has a similar magnitude to the solar neutrino mass splitting  $\approx 8meV$ , has already led to some neutrino theories of dark energy including most popularly mass varying neutrino theory, in which the scalar field (named the accelaron) giving mass to the neutrinos acts as the attractive potential leading to the dark energy repulsion. We find in general the scalar field theories have ad hoc potentials that cannot be justified from particle physics alone.

In the section we show that our axial force together with the plasma relic neutrinos left over from the big bang, can give rise to the dark energy of the universe. Our analysis is necessary over simplified, standard electromagnetic plasma is already very complex, and in the neutrino case we have the addition problem of quantum effects such as Pauli pressure, which we will must approximate. Naively one would expect a the relativistic plasma of neutrinos to behave like an electron, positron plasma, decaying as  $a^{-4}$ , where  $a$  is the cosmic expansion scalar factor, until the temperature falls to the mass of the neutrino, at which point they fall out of equilibrium and annihilate, leaving

Figure 6: Neutrino pair production on a neutrino, 4 possible ways



just an addition axi-photon background radiation. However in a plasma there is a plasma frequency below which the axi-photons are strongly absorbed by the neutrinos. Because the neutrino is so light this plasma frequency is very high, high enough we believe, to prevent the annihilation of the neutrinos to photons. Instead until the temperature falls as low as the plasma frequency, all the axi-photons convert to neutrinos. The energy density will now fall as  $a^{-3}$ , like matter instead of radiation. But any attractive potential will slow the decay even further leading to  $\omega < 0$ . Additional effects will also prevent the neutrino annihilation, as we saw in our section on neutrino interactions, the low mass left anti-right neutrinos can only annihilate into 3 axi-photon suppressing the cross-section to  $\alpha_a^{3/2}$ , and the neutrino pair produce should have a cross section approximately 4 times bigger. See the attached figures.

There are 3 different types of interaction in the neutrinos that may lead to an attractive potential in the plasma. There are the charge-charge potential, the spin magnetic field potential and the spin-spin potential.

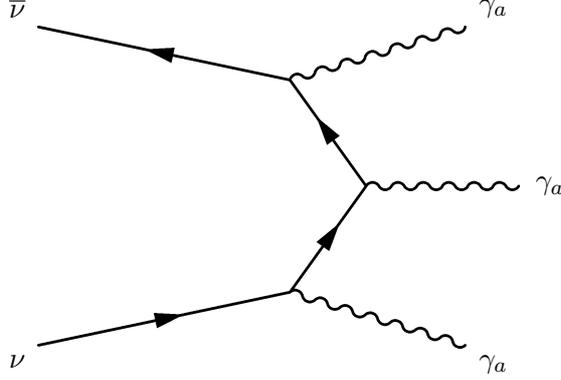
$$U_{cc} = M_{cc}n \frac{q_a^2}{4\pi\epsilon_a} \left\langle \frac{1}{r} \right\rangle \quad (123)$$

$$U_{ms} = M_{ms}n \frac{q^2}{4\pi\epsilon_a} \frac{\hbar}{2mc} \left\langle \frac{\gamma\beta}{r^2} \right\rangle \quad (124)$$

$$U_{ss} = M_{ss}n \frac{1}{4\pi\epsilon_a} \left( \frac{q_a\hbar}{2mc} \right)^2 \left\langle \frac{1}{r^3} \right\rangle \quad (125)$$

Where  $n$  is the number density and the  $M$  factor represent the configuration of the particles, and in particular  $M_{cc}$  is the Madelung constant.

Figure 7: Neutrino annihilation is only possible by 3 axi-photon emission



We will rewrite the above using the dimensionless force constant  $\alpha_a$  and the compton wavelength of the particle  $\lambda_c = h/mc$ . We are interested in densities where the potential energy is near the mass energy of neutrinos. As it turns out this first occurs at the density of  $n\lambda_c^3\alpha = 4$  for the spin-spin interaction, and at much higher densities ( $n_{ms}\lambda_c^3\alpha^3 = 1$  and  $n_{cc}\lambda_c^3\alpha^2 = 2$ ) for the coulomb and magnetic interactions. Hence we are interested in the spin-spin interaction only.

In the text above we claimed that the plasma frequency, which gives the effective mass of the axi-photon, will be high enough to prevent the neutrino anihilating into low energy axi-photon. At what density does this occur? In fact we have.

$$E_p^2 = h^2\omega^2 = \frac{h^2q_a^2n}{m\epsilon_a} = \frac{nch^3\alpha_a}{2m} \quad (126)$$

Putting  $E_p > mc^2$  we have

$$n > \frac{2m^3c^3}{h^3\alpha_a}, \quad \implies, \quad n\lambda_c^3 > \frac{2}{\alpha_a} \quad (127)$$

This is just half the density at which the spin-spin energy becomes equal to the mass energy of the particles. We should note however at these densities the overall energy of the plasma still does not become negative due to the Pauli energy of the plasma. At relativistic energies, the Pauli energy of the highest momentum particle is.

$$E_p = \sqrt{m^2c^4 + h^2c^2(3\pi^2n)^{2/3}} - mc^2 \quad (128)$$

Let us now consider the effect of the expansion of space on such a fluid. Introducing the Hubble constant,  $H$ , as the rate of increase of the seperation

of particles divided by there current seperation.

$$H = \frac{1}{r} \frac{dr}{dt} \quad , \implies \quad r = r_0 e^{Ht} \quad (129)$$

Let us introduce the scale factor  $a(t) = r/r_0$ , a neutrino number density  $n = n_0 a^{-3}$

Then

$$a = e^{Ht} \quad , \quad \frac{da}{dt} = H e^{Ht} = H a \quad (130)$$

and,

$$\frac{\partial n}{\partial t} = -3A a^{-4} \frac{da}{dt} = -3nH \quad (131)$$

The neutrinos and anti-neutrinos in our fluid will of course attract each other, due primary to the spin-spin interaction as descussed above.

$$U = -M_{ss} \cdot \alpha \frac{\lambda_c^2}{4} n^2 \hbar c \quad (132)$$

Let,

$$A = -M_{ss} \alpha \frac{\lambda_c^2}{4} \hbar c \quad (133)$$

Then,

$$U = -A n^2 \quad (134)$$

The effect of the expansion of space upon the potential energy, will be to introduce extra energy into to system at a rate:

$$\frac{\partial U}{\partial t} = -2A \frac{\partial n}{\partial t} = 6A n^2 H \quad (135)$$

What is going to happen to the extra energy?. We have argued above that the fluid is in an equilibrium of pair production. Each particle having a average kinetic energy of  $\chi m c^2$ . With  $\chi$  depending upon the Pauli energy. So any energy introduced into the system will pair produces new particles.

$$\frac{\partial n_p}{\partial t} = \frac{1}{2(1+\chi)m_\nu c^2} \frac{\partial U}{\partial t} = 6n^2 \frac{AH}{(1+\chi)m_\nu c^2} \quad (136)$$

Let,

$$D = \frac{A}{(1+\chi)m_\nu c^2} \quad (137)$$

The total change in density per unit unit, is the sum of the two effects.

$$\frac{dn}{dt} = -3nH + 6n^2 D H \quad (138)$$

This equation has an critical point at  $n = 1/2D$  providing the density is greater than this, the density will increase with the expansion of the universe,

otherwise it will reduce with the expansion of the universe. Putting  $D$  back into the above we find that, the density increases for

$$n > \frac{M_{ss}\lambda^3}{2\alpha(1+\chi)} \quad (139)$$

The above equation runs away to infinite density above our critical density. This cannot happen in reality, but what effect will prevent this from happening? We believe the rise in density will halt when Pauli energy reaches the next heaviest neutrino mass eigenstate at around  $8meV/c^2$ . At this point some of the neutrino will convert to the heavier state, and will produce a much lower spin-spin interaction. This gives a number density of.

$$hc(3\pi^2n)^{1/3} = 8meV \implies n = 1.4 \times 10^{13}m^{-3} \quad (140)$$

Putting  $nh^3c^3\alpha_a/4 > m^3$  then leads to

$$m_\nu < 0.22meV \quad (141)$$

Thus we believe that with a lightest neutrino with mass around  $0.2meV$  feeling the spin-spin interaction due to our axial force is able to produce the dark energy of the universe. In fact our model gives a total energy of

$$U_t = nmc^2 \left[ 1 - nM_{ss} \frac{h^3}{m^3c^3} \frac{\alpha_a}{4} + \frac{3}{5} \frac{8meV}{mc^2} \right] \quad (142)$$

$U$  is the total dark energy density, which should equal around  $(2meV)^4 \approx 2 \times 10^9 eV/m^3$ . If  $M_{ss}$  is taken to be 1, then this gives the correct energy density if  $m_\nu = 0.14meV$ . Thus we see that very reasonable parameters give the correct value for the dark energy of the universe.

Our above analysis is very rough, a more thorough analysis should calculate the exact spin-spin interaction between the neutrinos in a plasma at each particular density, this would likely require a lattice or Monte Carlo simulation. We also have not correctly calculated the  $\omega$  factor for our theory. This depends on what percentage of the plasmas energy is restored from the interaction potential after a infinitesimal expansion of the universe. We do not believe our model is capable of producing phantom ( $\omega < -1$ ) energy nor an analog of the cosmological constant ( $\omega = -1$ ), however  $\omega$  close to  $-1$  seems possible. With such an  $\omega$  the dark energy density will slowly deplete until it falls below our critical level given by equation (127), after which the neutrinos will annihilate down to axi-photons leaving just a new (but hotter) cosmic background radiation (CBR).

In our model the acceleration of the universe does not begin until late in the evolution of the cosmos. In particular assuming that the neutrino and axi-photon mixture, begins at the same temperature at the CBR and cools as

$a^{-4}$ . Then the acceleration of the universe begins when  $T = 8meV = 93K$ , since the temperature is now  $2.7K$ , this means that the acceleration started around  $z = 2.4$ . Slowing the decay of the neutrino energy or increasing its original quantity could make the acceleration start closer to modern times, but we see nothing that could have made it start earlier. Thus our model makes the following definite predications:  $\omega < -1$  the exact figure is calculable but not yet calculated. Given a neutrino of mass  $8\text{ meV}$  we predict neutrinos the lightest neutrino has a mass of  $0.14meV$ . Finally and most accessibly to observation we predict that acceleration started somewhen a little later than  $z = 2.4$ . The acceleration is to end sometime in the future as a gentle decay to a new CBR.

### 14.3 Supernovae

Simulations of supernovae find a problem, there often does not seem to be quite enough energy or force produced to destroy the star. Adding the axial force to the mix does not significantly increase the pressure from a single neutrino because the force is weak at high momenta. However as we saw in our section on axial symmetry restoration: Neutrons normally we have a strong repulsive pressure due the axial force. A state change needs to occur for axially neutral neutral neutronium to form. Thus core collapse will happen in two stages, first pressure converts protons and electrons to neutrons emitting neutrinos. Then the half the neutrons need to reverse there axial charge to form axially neutral neutronium. This second change will release anti-neutrinos and axi-photons. This two stage process will release more energy and pressurising high speed particles than the normal 1-stage process. At present we do not know enough to calculate this process. It need a good model of axial symmetry restoration, the full Higgs potential for quark masses, and a lot of simulation. However we can make one clear prediction, a burst of anti-neutrinos should be emitted in the early stage core collapse as well as matter-neutrino. In the standard picture of stella core collapse, there is first a burst (1-2 seconds) of neutrinos from the neutronisation of the matter in core. This is then followed by a slower thermal emission of pair created neutrinos and anti-neutrinos lasting (10-20) seconds. In our picture there will be anti-neutrinos emitted as well a neutrino during the neutronisation phase.

Only one event supernova event as thus far had it neutrino emission measured. In 1987, SN1987A exploded in the large magelenic cloud some hundred thousand light years away. At time three neutrino experiments picked up a total of 24 anti-neutrinos from the explosion. Unfortunately the only experiments running used water as a target and as such they could only detect anti-neutrinos. The signal from the measurements was clear however, first a quick burst of anti-neutrinos lasting a few seconds followed by a slower hump of anti-neutrinos lasting up to 20 seconds. Our model

predicts the initial burst to contain anti-neutrinos, the standard model does not seem to. Thus we can mark up a slight victory at the axial force model, but we really require a supernova somewhere in our galaxy for a clear signal.

A second piece of evidence for a second neutrino burst in supernova due to conversion to an axial neutral state, comes from details of the nucleosynthesis of heavy elements in supernova explosions. Heavy elements are thought to be formed by the r-process, rapid absorption of neutrons in expanding outer shell of the SN. However [45] it does not seem that enough neutrons are present in a standard SN explosions. Our process leads to an additional neutron flux, due to the absorption of anti-neutrinos by the protons of any hydrogen gas in the supernova's exterior. Thus the process below, (where El stands of any heavy element),

$$\begin{cases} n + n \rightarrow N + n + \bar{\nu} & \text{(In the core)} \\ \bar{\nu} + p \rightarrow n + e^- & \text{(In the ejected mass)} \\ n + {}_ZEl^w \rightarrow {}_ZEl^{w+1} + \gamma & \text{(r-process)} \end{cases}$$

should lead to an significantly enhanced neutron projects and r-process rates. Of course this is very qualitative and we need to confirm results of the mechanism in computer simulations of supernova.

## 15 The Tajmar experiment

In 2005-6 Tajmar et al [32] performed an experiment upon a rotating superconducting Niobium  $Nb_{41}^{93}$  ring. Accelerometers where placed next to the ring. It was observed that when the ring was angularly accelerated the accelerometer measured a force trying to rotate them in the opposite sense to the acceleration of the ring. This force was only detected when the ring was superconducting. It was shown that electric and magnetic field could not be the source of the effect. Tajmar tried to explain this force as due to the a gravimagnetic effect. We do not believe that this a possible explanation. Firstly the force due to gravity would be some  $10^{23}$  times smaller than the measured effect. Secondly gravimagnetism normally acts around rotating objects to drag spacetime in the same direction as the sense of the rotation. So the gravimagnetic effect is in the wrong sense as well as the wrong size.

We believe the Tajmar effect can be better explained by the axial force. We explain the result as due to an axi-magnetic field is generated due to the drag of the superconducting Cooper electron pairs on the left handed neutrinos in the superconductor, which occurs via the weak nuclear force. Rotating a superconductor produces a magnetic field, via the London moment which strength:

$$\vec{B} = -\frac{2m}{e}\vec{\omega} \quad (143)$$

This leads to a measurement of the mass of a Cooper Pair [33] which in theory is  $m_c/2m_e = 0.999992$  but was experimentally observed as 1.000084. We take the additional inertia of the Cooper pairs as due to their dragging effect upon the neutrino fluid inside the superconductor. This leads to aximagnetic field being produced by a rotating superconductor as well as an magnetic field. Then during acceleration, the aximagnetic field will increase, inducing an radial axi-electric field upon the silicon plates in the accelerometer driving a neutrino current. Finally the motor effect, produces a force on the accelerometers due to the induced neutrino current interacting with the axi-magnetic field.

$$F = B_a \times (II_a) = B_a \times (r_1 - r_2) \frac{\partial B_a}{\partial t} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \frac{1}{R_a} \approx \frac{l^2}{R_a} \frac{B_a}{r^2} \times \frac{\partial B_a}{\partial t} \quad (144)$$

Where  $R_a$  is the resistance to neutrino flow in the silicon sensor, and  $r_1$  and  $r_2$  are the distances from the ring. Since no complete circuit exists the force can only be produced until the (axi) capacitance of the silicon plate is full charged by the current thus.

$$\int F dt \approx \max \left( \frac{C_a}{R_a} \frac{\partial B_a}{\partial t}, \frac{\Delta B_a}{R_a} \right) B_a \quad (145)$$

Approximately depending only upon the neutrino capacitance and resistivity of the accelerometer plates. Without detailed material knowledge of the accelerometer it is difficult to produce more complete quantitative calculation of the measured acceleration. The acceleration due to our theory, like that of Tajmar's graviphoton theory, depends linearly upon the Cooper pair density, and Cooper pair mass excess. But we also predict oscillation to be observed in the acceleration measurements, due to an restoring current in the sensor which occurs once the superconducting ring reaches steady velocity. This also seems to be observed in Tajmar's graphs.

To experimentally test the difference between our explanation of the Tajmar effect, and his own gravimagnetic explanation, we can firstly try to shield the aximagnetic force. Tajmar experiment was performed in a vacuum, air should shield the axial force as should most materials except carbon-12. The sensors used were made of silicon, we predict that if isotopically pure silicon-32 was used, no signal would be measured. However if pure silicon-33 was used the signal should be stronger. Finally with a source of a varying axi-magnetic field, all manner of experiments are possible, which should be able to show the equivalent of maxwells equations for the axial force.

## 16 Other possible effects of the axial force

In this section we place a few speculations as to problems that may be solved by our axial force, that we have not had time to compute or test

quantatively.

Black hole jets are also astrophysical problem, most models explain it with the magnetic field of the accretion disk, but we see no obvious reason why the electric current should be strong in accretion disk plasma. Instead we suggest that the infalling matter carries a net axial charge. The black hole will then need to expel this axial charge, by neutrino pair production. But only at the poles can these neutrinos escape, and the axial force will then drag matter at the poles along with it. Thusly we show how black holes can both suck and blow at the same time.

In the early universe the first stars seem to have formed some 200 million years after the big bang. This seems too early for most theories to explain. The major problem being that neutral atomic hydrogen atoms cannot lose energy easily and neither is it quick for them to combine into molecular hydrogen without the presence of heavier atoms. Although our axial force predicts long-range repulsive force between protons, its possible to see that in the presence of an neutrino background this becomes attractive at sufficiently distance range, it also provides an additional way for a gas of atomic hydrogen to lose energy. Thus the axial force should aid the collapse of matter to form stars and galaxies.

A thougher reader might note that although we have talked about and attempted to solve many of the mysteries of contemporary physics, we have played little attention to the problem of explaining the dark matter content of the universe. In fact our axial force does not seem to lead to any obvious new dark matter candidates. However it does remove a few old ones. The main warm dark matter candidates, sterile neutrinos and axions are both contridicated by our axial force theory. Axions are now not needed to solve the QCD theta problem, and the right handed neutrinos our theory predicts to decay very rapidly. Astrophysical observations seem however to have already ruled out warm dark matter, and so it is of little loss that our theory rules out the main WDM candidates particle.

Cold Dark Matter is on the contrary is the current standard choice of the extra matter in the universe. CDM has its own problems including making the cores of galaxies and globular clusters to cuspy in there density profiles. The standard candidate particle for CDM, is the light supersymmetric particle, the LSP. We have not attempted to made our axial force theory supersymmetric, but there does not seem to be any obvious difficulty in doing so. However our axial force may yet cause problems for CDM. Supersymmetric theory needed to introduce r-parity, a conversed count of the number of supersymmetric particles, in order to keep the proton stablised. This then leads to the stablity of the lightist supersymmetric particle. Our axial force, automatically lead to baryon and lepton number conservation just due the seperate conservation of the electric and axial charges. Thus a supersymmetric theory with the axial force, doesn't need r-parity or a stable LSP. The axial force however does not rule out such a particle. But

perhaps our theory such push researchers towards other candidate CDM particles, of perhaps to mirror [21] type dark matter.

## 17 How can such a large force have been missed

Because our force strength  $\alpha_a$  is relatively large it become an important consideration that its effects do not break the of results years of physical experiments. We are aided by the fact that electrons and other charged leptons do not interact with the force and so atomic and chemical physics measurements should be largely oblivious to our new force. As we saw earlier the force leaves the weak force slightly weaker than expected. We have also seen the axial force neutrino scattering is strongly suppressed due to the fact angular momentum conservation at a vertex means that to emit a spin one axiphoton, the neutrino's spin must flip, this is only possible if neutrinos linear momentum also reverses, thus only complete backscattering is allowed. Such backscattering is rarely measured in experiments and is a small contribution to the total scattering phase space. This also means that the axial force between neutrinos is weak at high speeds, but much larger at low velocities, allowing it to have a strong effect cosmic neutrino background now, but little effect at the time of big bang.

In hadronic experiments the axial force will of cause be swamped by the large strong force, the error in QCD calculations being large enough to hide a multitude of effects.

The axiphoton itself presents a problem. It may be produced in any particle collisions involving quarks or neutrinos (which is most of them), at a rate of about 1/90 of photon production in the case of quarks. It will not be absorbed by electrons in experimental aparatus, but will interact with nuclei. Thus it will look like an strongly penetrating gamma ray in many experiments. It will be detected in scintallators and calorimeters, and by nuclear recoil in bubble chambers, but no electron position pair production will occur. Thus a signiture of the axial force will be an anomously low pair production and compton scattering rates with gamma rays from certain experiments. We doubt that this has yet been systematically checked for, but it should be relatively easy to do so in data from old experiments.

On the large scale, matter will be axially neutral, there being two ways that this is archived, Firstly nuclides with identical proton and neutron number will have zero axial charge. Whilst, with differing nucleon numbers, a diffuse gas of neutrino or antineutrinos will be needed to balance the axial charge on the nucleii. Since the neutrino mass is so small, the potential of the axial field is easily enough to pair produce neutrinos and then expel the type (normal or anti) with the wrong charge. Such material will screen the axial electric and magnetic fields very efficiently. With axially netural matter and in the vacuum, there will still exist the cosmic neutrino background with

some overdensity from thermal pair production from near by matter. Thus at atomicly medium or long range no net charge will give rise to any force be that is easierly detectible.

Analogously, to the casmir force between neutral matter due to the electric field, there would be a casmir force due to the axial field, however since casmir forces depend on the force constant to the fifth power, this would be completely swamped by the EM casmir force. Hence we can be clear that none of the fifth force gravitational experiments reviewed by Adelberger, Heckel & Nelson [5] would be sensitive to our axial force. Fischbach and Talmadge [6] have reviewed 10 unreplicated experiments that did show evidence for a fifth force. Could any of them be our axial force. If the earth had a small net axielectric charge or more likely aximagnetism, then perhaps the Kaon data mentioned might be due the axial force, in the other cases the chance of a material test particle haveing picked up a net axial charge could explain the results. But none of those results provide strong evidence of our force.

In condensed matter, isotopes with the axial interactions will make changes to the properties of elements and compounds. In particular we would expect a slight increase in the boiling points of isotopes with net axial charges. Thermal properties will also be effected, matter make with isotopes with axially charged nuclei should show larger thermal conductivity due axiphotons carrying some of the thermal radiation. Remarkably isotopic effects can be quite strong in condensed matter experiments. For example [4] in Boron carbide, changing the carbon from  $^{12}C$  to  $^{13}C$  increased conductivity by around 10% but changing the boron from  $^{10}B$  to  $^{11}B$  reduced it around 16%, such effects might be best explained if unpaired neutrons in isotopes feel the axial force. With our charge assignment of  $\pm\frac{1}{2}$  on the nucleon the axial force at first glance explains the carbon substution but not the boron effect. Such isotopic effects are difficult to explain in conventional physics but perhaps the remoteness of condensed matter from particle physics has prevented a clamor for solutions to such problems.

High Temperature Supercoductors also show strong isotopic effects, we speculate that the neutrino background field interacts with electrons in the superconductor. When the neutrino background is dense, it may distrubt Cooper pair production. Conversely as we saw in our discussion of the [32] Tajmar experiment, when the neutrino background is sparse, Cooper pairs may lock the neutrino field in a fixed position, preventing it from screening the aximagnetic force. If this is the case then the highest temperature superconductors will occur in an isotopic mix with the least axial charge on the nuclei.

## 18 Conclusions and belief

How much then should we believe in this axial force. To my eye, the fact that it is derivable directly from gauge invariance, and automatically makes the weak force handed, looks beautiful. The standard model significantly simplifies upon adding the axial field. The axion and majoron go from being necessary components of the model to being useless and indeed banned states. The global lepton and baryon number symmetries also become redundant, their conservation being enforced by the necessity of conserving both electric and axial charge at the same time.

After this though it gets murkier, we are forced to add heavy quark states with opposite axial charge, and further the mass type (Dirac versus Majorana) of the quarks needs to change at high pressures in order to have stable neutron stars. That is somewhat ugly and we cannot claim to have proved the mechanism is possible. This does predict a early anti-neutrino burst from a supernova, a prediction that seems to be born out in the SN1987a events. This same procedure does seem to provide an possible explanation of the mystery  $\sigma(555)$  meson states, and fits nicely into the  $E6$ ,  $E7$  and  $E8$  exceptional groups favoured by string theory.

To neutralise the axial charge, we are forced to introduce a dense sea of neutrinos in ordinary matter, and have to have low mass right handed neutrino states to prevent the Fermi energy getting out of hand. These screen the axial force very effectively making it difficult to detect. Bizarrely we seem to have predicted, that right handed neutrinos need to be created when matter gets denser (e.g. during the condensation of water), and decay when as matter gets sparser (e.g. during boiling), these changes would be rapid and need to be hidden in the latent heat of materials. Having made this leap of faith, we get some genuine physical predictions including the transition of the sun chromosphere to its corona.

If it exists that axial force will be a ubiquitous new component of the universe, causing or explaining phenomena at every scale. We have found two experiments, the Miniboone experiment and the Tajmar experiment that seem well explained by the axial force. We also have an explanation of Dark energy, which is of vital importance to the universe. Finally of course we have explained why the neutrino is always left handed. I believe this should be enough to stimulate the physics community to test and refine the theory of the axial force. And may perhaps be the additional boost needed to move from the standard model to more unified field.

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## A Appendix A, an alternative neutrino wave equations

The equation for the left handed neutrino is,

$$\left( i \frac{\partial}{\partial t} - i \sigma'_j \frac{\partial}{\partial x_j} - m_l R \right) \phi_l = 0. \quad (146)$$

which may be multiplied by its conjugate, (the hermitian conjugate plus reversing the sign of  $\sigma'$ ).

$$\phi_l^\dagger \left( i \frac{\partial}{\partial t} + i \sigma'_j \frac{\partial}{\partial x_j} - m_l R \right) = 0. \quad (147)$$

This will regenerate the Klein Gordon equation provided, that

$$\sigma' = \sigma'^\dagger \quad \sigma'^2 = 1 \quad R = R^\dagger \quad R^2 = 1 \quad \sigma R = -R\sigma \quad (148)$$

In 4 dimensions total, the possible solutions are.

$$\sigma'_i = S \otimes \sigma_i, \quad R = R' \otimes I_2 \quad S, R' \in (\sigma_1, \sigma_2, \sigma_3) \quad S \neq R \quad (149)$$

The correct current conservation equation will be produced by  $\phi_a^\dagger(146) - (146)^\dagger \phi_a$  provided that  $R = R^\dagger$

We will run through each of the possibilities in order to show them to be equivalent.

### A.1 Solution 1. $S = \sigma_1, R' = \sigma_2$

$$Eu_b - (\mathbf{p} \cdot \sigma - im)u_a = 0 \quad , \quad Eu_a - (\mathbf{p} \cdot \sigma + im)u_b = 0 \quad (150)$$

We have solutions:

$$v^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} s \\ \mathbf{p} \cdot \sigma s/E - im \cdot s/E \end{pmatrix} \quad , \quad v^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{p} \cdot \sigma s/E + im \cdot s/E \\ s \end{pmatrix} \quad , \quad (151)$$

The completeness relation is (with  $s_1 = (1, 0)^T$  and  $s_2 = (0, 1)^T$ ), and  $u$  labeled as in the main text

$$\sum_{s=1..4} u_s u_s^\dagger = I + \frac{\sigma' \cdot p - mcR}{E} \quad (152)$$

The helicity is,

$$h = \frac{p^2}{2E|p|} \quad (153)$$

And the current

$$J_i = \frac{q}{2} s^\dagger \frac{[\sigma_i \tilde{\sigma} \cdot \mathbf{p} + \tilde{\sigma} \cdot \mathbf{p} \sigma_i]}{E} s = \mathbf{v} \odot \mathbf{s} \quad (154)$$

**A.2 Solution 2.**  $S = \sigma_2, R' = \sigma_1$

$$Eu_b + (-i\mathbf{p}\cdot\sigma - m)u_a = 0 \quad , \quad Eu_a + i(\mathbf{p}\cdot\sigma - m)u_b = 0 \quad (155)$$

We have solutions:

$$v^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} s \\ ms/E - i\mathbf{p}\cdot\sigma s/E \end{pmatrix} \quad , \quad v^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} ms/E + i\mathbf{p}\cdot\sigma s/E \\ s \end{pmatrix} \quad , \quad (156)$$

The completeness relation is

$$\sum_{s=1..4} u_s u_s^\dagger = I + \frac{mR - \sigma' \cdot p}{E} \quad (157)$$

The helicity is,

$$h = \frac{p^2}{2E|p|} \quad (158)$$

And the current

$$h = \frac{p^2}{2|p|E^2}, \quad \phi = v^{(1)} \quad (159)$$

$$J_i = \frac{q}{2} s^\dagger \frac{[\sigma_i \tilde{\sigma} \cdot \mathbf{p} + \tilde{\sigma} \cdot \mathbf{p} \sigma_i]}{E} s = \mathbf{v} \odot \mathbf{s} \quad (160)$$

So this is almost identical to the first choice

**A.3 Solution 3.**  $S = \sigma_3, R' = \sigma_1$

$$(E - \mathbf{p}\cdot\sigma)u_a - mu_b = 0 \quad , \quad (E + \mathbf{p}\cdot\sigma)u_b - mu_a = 0 \quad (161)$$

We have solutions:

$$v^{(1)} = \frac{1}{N} \begin{pmatrix} s \\ Es/m - \mathbf{p}\cdot\sigma s/m \end{pmatrix} \quad , \quad v^{(2)} = \frac{1}{N} \begin{pmatrix} Es/m + \mathbf{p}\cdot\sigma s/m \\ s \end{pmatrix} \quad , \quad (162)$$

But the normalisation depends on the spin. So we can't easily use this choice.

**A.4 Solution 4.**  $S = \sigma_1, R' = \sigma_3$

$$(E - m)u_a - (\mathbf{p}\cdot\sigma)u_b = 0 \quad , \quad (E + m)u_b - (\mathbf{p}\cdot\sigma)u_a = 0 \quad (163)$$

We have solutions:

$$v^{(1)} = \frac{1}{N} \begin{pmatrix} s \\ \mathbf{p}\cdot\sigma s/(E + m) \end{pmatrix} \quad , \quad v^{(2)} = \frac{1}{N} \begin{pmatrix} E\mathbf{p}\cdot\sigma s/(E - m) \\ s \end{pmatrix} \quad , \quad (164)$$

Again the normalisation depends on the spin. Lets ignore this one.

**A.5 Solution 5.**  $S = \sigma_3, R' = \sigma_2$

$$(E - \mathbf{p} \cdot \boldsymbol{\sigma})u_a + imu_b = 0 \quad , \quad (E + \mathbf{p} \cdot \boldsymbol{\sigma})u_b - imu_a = 0 \quad (165)$$

We have solutions:

$$v^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} s \\ i(E - \mathbf{p} \cdot \boldsymbol{\sigma})s/m \end{pmatrix} \quad , \quad v^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i(E + \mathbf{p} \cdot \boldsymbol{\sigma})s/m \\ s \end{pmatrix} \quad , \quad (166)$$

Another case, where normalisation depends on the spin.

**A.6 Solution 6.**  $S = \sigma_2, R' = \sigma_3$

$$(E - m)u_a + (i\mathbf{p} \cdot \boldsymbol{\sigma})u_b = 0 \quad , \quad (E + m)u_b - i(\mathbf{p} \cdot \boldsymbol{\sigma})u_a = 0 \quad (167)$$

We have solutions:

$$v^{(1)} = \frac{1}{\sqrt{2E}} \begin{pmatrix} s \\ -i\mathbf{p} \cdot \boldsymbol{\sigma} s / (E + m) \end{pmatrix} \quad (168)$$

$$v^{(2)} = \frac{1}{\sqrt{2E}} \begin{pmatrix} i\mathbf{p} \cdot \boldsymbol{\sigma} s / (m - E) \\ s \end{pmatrix} \quad (169)$$

One of the solutions disappears at  $p = 0$ , while for the antiparticle (reverse  $E$  or  $m$  not both) the other solution is the one that disappears.

The completeness relation is:

$$\sum_{s=1..4} u_s u_s^\dagger = I - \frac{\boldsymbol{\sigma}' \cdot \mathbf{p} + mR}{E} \quad (170)$$

The helicity is for the  $v^{(1)}$  solution

$$h^{(1)} = -\frac{p^2}{2|p|E} \quad (171)$$

And for the  $v^{(2)}$  solution

$$h^{(2)} = \frac{p^2}{2|p|E} \quad (172)$$

And the current

$$h = \frac{p^2}{2|p|E^2}, \quad \phi = v^{(1)} \quad (173)$$

The current for the  $v^{(1)}$  solution is

$$J_i = -\frac{q}{2} s^\dagger \frac{[\sigma_i \tilde{\boldsymbol{\sigma}} \cdot \mathbf{p} + \tilde{\boldsymbol{\sigma}} \cdot \mathbf{p} \sigma_i]}{E} s = -\mathbf{v} \odot \mathbf{s} \quad (174)$$

The current for the  $v^{(2)}$  solution is

$$J_i = \frac{q}{2} s^\dagger \frac{[\sigma_i \tilde{\boldsymbol{\sigma}} \cdot \mathbf{p} + \tilde{\boldsymbol{\sigma}} \cdot \mathbf{p} \sigma_i]}{E} s = \mathbf{v} \odot \mathbf{s} \quad (175)$$

Just when we thought we had no different workable solutions, we find another sets of states that work, reassuringly we get the same current and helicity for these state, up to a minus sign. These sets are a different choice on how to handle the particle and anti-particle states. With these choice of states, we can clearly see how while we lose a spin choice at  $v = 0$ , the helicity still defined there, as is the internal spin choice.